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LAMINAR CONVECTIVE HEAT TRANSFER
IN THE ENTRANCE REGION BETWEEN
PARALLEL FLAT PLATES

D. D. LUNDBERG

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LAMINAR CONVECTIVE HEAT TRANSFER
IN THE ENTRANCE REGION BETWEEN PARALLEL FLAT PLATES

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D. D. Lundberg



LAMINAR CONVECTIVE HEAT TRANSFER
IN THE ENTRANCE REGION BETWEEN PARALLEL FLAT PLATES

by

D. D. Lundberg

Captain, United States Marine Corps

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
AERONAUTICAL ENGINEERING

United States Naval Postgraduate School
Monterey, California

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This work is accepted as fulfilling
the thesis requirements for the degree of

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IN

AERONAUTICAL ENGINEERING

from the

United States Naval Postgraduate School

ACKNOWLEDGEMENT

The author wishes to express his gratitude to Dr. James A. Miller, Associate Professor, Department of Aeronautical Engineering, for his assistance, and interest. Dr. Miller suggested the project and provided his paper on the subject, which proved to be a valuable reference. In addition, Professor Miller's excellent lectures on boundary layer theory, and heat and mass transfer were found to be highly informative and stimulating.

MEMORANDUM

TO : The President

FROM : The Vice President

SUBJECT: [Illegible]

[Illegible text follows, consisting of several paragraphs of text that are too blurry to transcribe accurately.]

ABSTRACT

Heat transfer rates for laminar, convective heat transfer in the entrance region between parallel plates are investigated. The hydrodynamic solution due to Bodia $\overline{[2]}$ was used in the solution of the energy equation in finite difference form on a digital computer. The thermal boundary conditions include: constant heat input, constant wall temperature, one wall constant temperature and one wall insulated, and constant but different wall temperatures on the upper and lower walls.

The approximate, integral methods of Siegel and Sparrow $\overline{[6]}$, $\overline{[7]}$ produce results that are in close agreement with the solutions in this analysis for the constant heat input and constant wall temperature cases.

The scope of the finite difference solution is limited to a narrow range of Prandtl numbers near unity, due to the small grid sizes required for convergence at small Prandtl numbers and to the overly low transfer rates indicated near the entrance for high Prandtl numbers, which is a result of the "finite starting length."

REPORT

The following report was prepared by the committee on the subject of the proposed new law, and is submitted to the assembly for their consideration. The committee has the honor to acknowledge the assistance of the various departments of the government, and the cooperation of the various officers and employees of the same, in the preparation of this report. The committee also wishes to express its appreciation to the various members of the assembly, who have assisted it in its work, and to the various officers and employees of the assembly, who have assisted it in its work. The committee has the honor to acknowledge the assistance of the various departments of the government, and the cooperation of the various officers and employees of the same, in the preparation of this report. The committee also wishes to express its appreciation to the various members of the assembly, who have assisted it in its work, and to the various officers and employees of the assembly, who have assisted it in its work.

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No.	Description	Amount
1	To balance forward	100.00
2	By cash	50.00
3	By cash	25.00
4	By cash	25.00
5	By cash	25.00
6	By cash	25.00
7	By cash	25.00
8	By cash	25.00
9	By cash	25.00
10	By cash	25.00
11	By cash	25.00
12	By cash	25.00
13	By cash	25.00
14	By cash	25.00
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16	By cash	25.00
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18	By cash	25.00
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23	By cash	25.00
24	By cash	25.00
25	By cash	25.00
26	By cash	25.00
27	By cash	25.00
28	By cash	25.00
29	By cash	25.00
30	By cash	25.00

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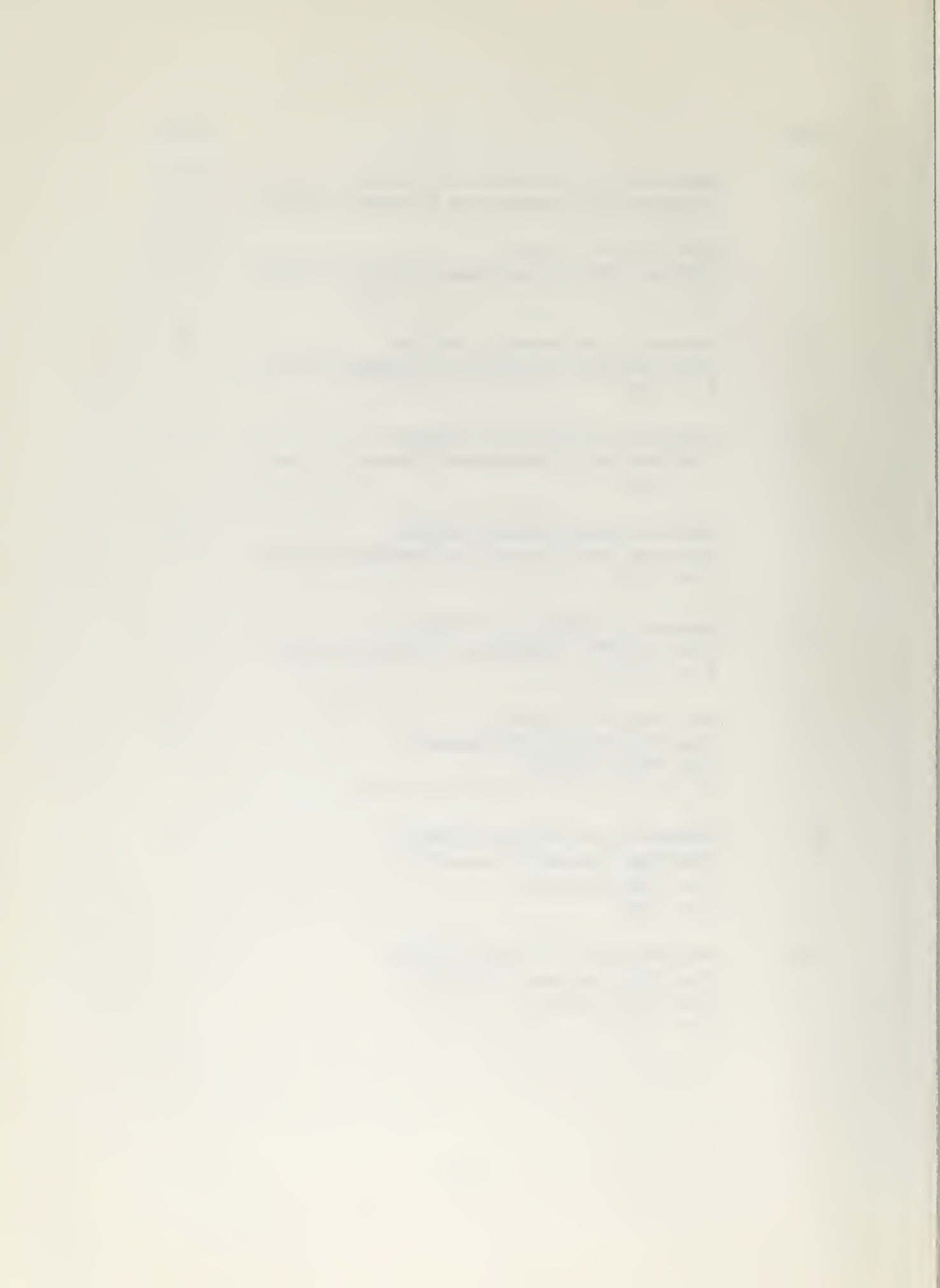


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1	Jan 1	Balance	100.00
2	Jan 2	Jan 3	100.00
3	Jan 4	Jan 5	100.00
4	Jan 6	Jan 7	100.00
5	Jan 8	Jan 9	100.00
6	Jan 10	Jan 11	100.00
7	Jan 12	Jan 13	100.00
8	Jan 14	Jan 15	100.00
9	Jan 16	Jan 17	100.00
10	Jan 18	Jan 19	100.00
11	Jan 20	Jan 21	100.00
12	Jan 22	Jan 23	100.00
13	Jan 24	Jan 25	100.00
14	Jan 26	Jan 27	100.00
15	Jan 28	Jan 29	100.00
16	Jan 30	Jan 31	100.00
17	Feb 1	Feb 2	100.00
18	Feb 3	Feb 4	100.00
19	Feb 5	Feb 6	100.00
20	Feb 7	Feb 8	100.00
21	Feb 9	Feb 10	100.00
22	Feb 11	Feb 12	100.00
23	Feb 13	Feb 14	100.00
24	Feb 15	Feb 16	100.00
25	Feb 17	Feb 18	100.00
26	Feb 19	Feb 20	100.00
27	Feb 21	Feb 22	100.00
28	Feb 23	Feb 24	100.00
29	Feb 25	Feb 26	100.00
30	Feb 27	Feb 28	100.00
31	Feb 29	Feb 30	100.00
32	Mar 1	Mar 2	100.00
33	Mar 3	Mar 4	100.00
34	Mar 5	Mar 6	100.00
35	Mar 7	Mar 8	100.00
36	Mar 9	Mar 10	100.00
37	Mar 11	Mar 12	100.00
38	Mar 13	Mar 14	100.00
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42	Mar 21	Mar 22	100.00
43	Mar 23	Mar 24	100.00
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46	Mar 29	Mar 30	100.00
47	Mar 31	Mar 31	100.00

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TABLE OF SYMBOLS

English Letter Symbols

c	Specific heat
d	Half the distance of separation of the parallel flat plates
D	Hydraulic diameter $D = 4d$
h	Unit conductance for convection heat transfer
k	Unit thermal conductivity
L	Dimensionless length parameter, $\frac{x/D}{Re_D} = \frac{X}{16}$
q"	Heat flux per unit area, q/A
q*	Dimensionless heat flux, $\frac{qD}{Akt_o}$
t	Temperature
t _o	Initial fluid temperature
t _m	Mixed mean temperature
t _w	Wall temperature
t _{w1}	Lower wall temperature (asymmetric case)
t _{w2}	Upper wall temperature (asymmetric case)
T	Dimensionless temperature, t/t_o
T _w	Dimensionless wall temperature, t_w/t_1
T _m	Dimensionless mixed mean temperature, t_m/t_o
u	Axial velocity component

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u_o	Initial axial velocity component
U	Dimensionless axial velocity, u/u_o
v	Crosswise velocity component
v_c	Crosswise velocity component at centerline
v_o	Initial crosswise velocity component
V	Dimensionless crosswise velocity component, v/u_o
X	Dimensionless length parameter, $\frac{\nu x}{d^2 u_o}$
y	Normal coordinate
Y	Dimensionless normal coordinate, y/d

Greek Letter Symbols

ρ	Fluid density
μ	Fluid viscosity
ν	Kinematic viscosity, μ/ρ

Non-dimensional Groups

N_{u_x}	Local Nusselt number, $\frac{hD}{k}$
PR	Prandtl number, $\frac{\mu c_p}{k}$
Re_d	Reynolds number, based on half the gap width, $\frac{du_o}{\nu}$
Re_D	Reynolds number, based on hydraulic diameter, $\frac{Du_o}{\nu}$

I. INTRODUCTION

Efficient design of compact heat exchangers such as are employed for gas turbine regenerators, among other applications, requires maximum utilization of the high transfer rates available in the entrance region. In this region the velocity and temperature profiles of the entering fluid are undergoing rapid transition from their uniform distribution at entrance to the fully established distributions encountered farther downstream. The well known Graetz solution, in which only the temperature development is considered, does not yield adequate results when the velocity and temperature distributions are simultaneously developing. This is particularly true if the entrance region represents a considerable percentage of overall length and the two profiles develop at approximately the same rates. This situation is frequently encountered with gases where the Prandtl number is approximately one.

The purpose of the present analysis is to present a finite difference solution for the heat transfer rates between parallel flat plates for several different boundary conditions corresponding to various constant wall temperatures and constant heat input configurations. The approach to the problem was suggested by Miller $\overline{\angle 1}$, which is adapted from the numerical procedures applied to the entrance region of the circular tubes by Kays $\overline{\angle 2}$.

While an analysis such as the present one can hardly compete for simplicity with approximate treatments such as the Graetz analysis, or the several energy integral analyses commonly employed in engineering design, it does provide an exact solution for a limited number of cases which serves as a standard with which the accuracy of the various approximate methods may be assessed.

II. ANALYSIS

A. Governing Equations

The governing equations are the energy, momentum (Navier-Stokes) and continuity equations. In order to reduce the complexity and coupling of the equations, the following assumptions are made.

The flow is assumed to be:

1. steady, two-dimensional
2. laminar
3. incompressible

In addition it is assumed that:

1. thermal diffusivity (α) is constant
2. convective heat transfer is large compared to radiation, axial conduction, and viscous dissipation.

The governing equations in reduced form may then be expressed as:

Energy:

$$u \frac{\partial t}{\partial x} + v \frac{\partial t}{\partial y} = \alpha \frac{\partial^2 t}{\partial y^2} \quad (1)$$

Momentum:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \quad (2)$$

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

Due to the assumption of incompressibility the above equations are no longer coupled. The temperature development remains dependent upon the velocity profile, although dependence of velocity on temperature has been removed.

The boundary conditions associated with equations (1), (2), and (3) for flow between parallel flat plates with uniform velocity and temperature at entrance are:

1. At the entrance ($x = 0$)

$$t = t_o = C_1$$

$$u = u_o = C_2$$

$$v = v_o = 0$$

2. At the walls ($y = \pm d$)

$$v = v_w = 0$$

$$u = u_w = 0$$

$$t = t_w$$

3. On centerline

$$v = v_c = 0$$

4. Applied thermal boundary conditions

(1) symmetric cases (with respect to duct centerline)

a. constant wall temperature

$$t_w = C_3$$

b. constant heat input

$$\left(\frac{\partial t_w}{\partial y}\right)_{y=0} = C_4$$

(2) asymmetric cases

a. one wall constant temperature, one wall insulated

$$t_{w_1} = C_5$$

$$\left(\frac{\partial t_{w_2}}{\partial y}\right)_{y=0} = 0$$

b. constant wall temperature

$$t_{w_1} = C_6$$

$$t_{w_2} = C_7$$

where: $C_6 \neq C_7$

Solution of the energy equation requires that a hydrodynamic solution be first obtained.

B. Hydrodynamic Solution

Several hydrodynamic solutions for the entrance region are available. Solutions have been presented by Bodoia $\overline{\angle 3}$, Schlichting $\overline{\angle 4}$, Schiller $\overline{\angle 5}$, and Han $\overline{\angle 6}$. The approximate solutions of Schiller and Han have the advantage of being analytic, and therefore, less cumbersome to use. They are, however, inherently less accurate than the numerical solutions presented by Bodoia and Schlichting.

MEMORANDUM

TO : THE PRESIDENT

FROM : THE SECRETARY

SUBJECT: [Illegible]

DATE: [Illegible]

RE: [Illegible]

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Bodoia's solution was obtained by evaluation of the Prandtl boundary layer equations in finite-difference form. Velocity profiles were obtained at given streamwise locations by utilizing matrix methods to simultaneously solve a column array of momentum expressions normal to the flow.

A comparison of Schlichting's series solution and the Bodoia solution indicates that, in addition to the velocity gradient discontinuity in the Schlichting solution where the upstream and downstream solutions are joined, Schlichting's representation of the pressure gradient, based only on the centerline velocity, results in a core velocity gradient which is too large and a wall velocity gradient which is too small $\sqrt{3}$.

Comparisons of the dimensionless pressure decrement versus length due to Bodoia, Schlichting and Han are presented in Fig. 2. Velocity distribution comparisons between Bodoia and Schlichting, and Schlichting and Schiller, are shown in Figs. 3 and 4.

The Bodoia solution was adopted for the present analysis.

C. Energy Solution

Equations (1) and (3) may be written in dimensionless form by introducing the following dimensionless variables:

$$\begin{aligned} T &= t/t_o & X &= \frac{\nu x}{d^2 u_o} = \frac{x/d}{Re} & U &= u/u_o \\ Y &= y/d & V &= v/u_o \end{aligned}$$

In dimensionless form, the equations are then:

Energy:

$$U \frac{\partial T}{\partial X} + Re V \frac{\partial T}{\partial Y} = \frac{1}{Pr} \frac{\partial^2 T}{\partial Y^2} \quad (4)$$

Continuity:

$$\frac{\partial U}{\partial X} + Re \frac{\partial V}{\partial Y} = 0 \quad (5)$$

The following finite difference approximations are then introduced:

$$\frac{\partial T}{\partial X} = \frac{\Delta T}{\Delta X} = \frac{T_{x+1,y} - T_{x,y}}{\Delta X} \quad (6)$$

$$\frac{\partial T}{\partial Y} = \frac{\Delta T}{\Delta Y} = \frac{T_{x,y+1} - T_{x,y-1}}{2 \Delta Y} \quad (7)$$

$$\frac{\partial^2 T}{\partial X^2} = \frac{T_{x,y+1} - 2T_{x,y} + T_{x,y-1}}{\Delta Y^2} \quad (8)$$

$$\frac{\partial U}{\partial X} = \frac{\Delta U}{\Delta X} = \frac{U_{x+1,y} - U_{x-1,y}}{2 \Delta X} \quad (9)$$

$$\frac{\partial V}{\partial Y} = \frac{\Delta V}{\Delta Y} = \frac{V_{x,y} - V_{x,y-1}}{\Delta Y} \quad (10)$$

Note that (6) is a forward difference and (10) is a backward difference, whereas the remaining equations are of the central difference

type. These have been expressed in this manner for simplicity and convenience. The larger truncation error normally associated with the forward or backward difference is not significant here since practical grid dimensions require that:

$$\Delta Y \gg \Delta X$$

and, due to the nature of the flow:

$$\frac{\Delta U}{\Delta X} \gg \frac{\Delta V}{\Delta Y}$$

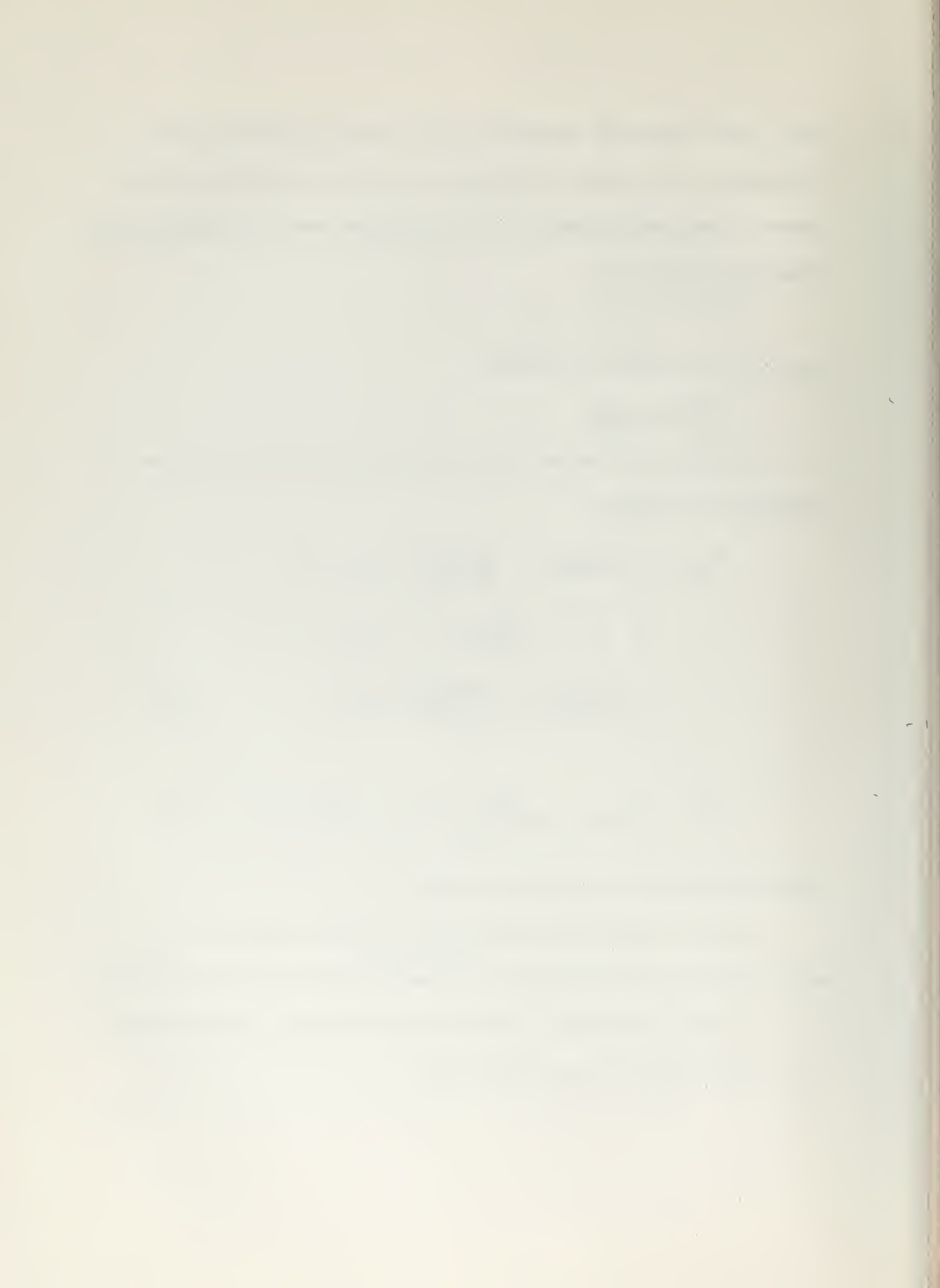
Introducing (6) through (10) into (4) and (5) and solving for the appropriate term yields:

$$\begin{aligned} T_{x+1,Y} = & \left[\frac{\Delta X}{U P R \Delta Y^2} + \frac{\Delta X R e V}{U^2 \Delta Y} \right] T_{x,Y-1} + \\ & \left[1 - \frac{2 \Delta X}{U P R \Delta Y^2} \right] T_{x,Y+1} + \\ & \left[\frac{\Delta X}{U P R \Delta Y^2} - \frac{\Delta X R e V}{U^2 \Delta Y} \right] T_{x,Y+1} \end{aligned} \quad (11)$$

$$V_{x,Y} = V_{x,Y-1} + \frac{\Delta Y}{R e \Delta X} \left[U_{x-1,Y} - U_{x+1,Y} \right] \quad (12)$$

which constitute the computing equations.

Note that the Reynolds number (R_e), although appearing in the above equations, remains only for convenience and could be eliminated. It is no longer a parameter in the set of equations having been included in the dimensionless length variable (X).



A mixed mean temperature (t_m) may be expressed in the following way:

$$t_m = \frac{\int c_p \rho u t dA}{\int c_p \rho u dA} \quad (13)$$

Removing the constants from within the integral, and simplifying, a dimensionless mixed mean temperature may be written in finite difference form:

$$T_m = \frac{\sum_{Y=0}^{Y=1} U_{x,Y} T_{x,Y}}{\sum_{Y=0}^{Y=1} U_{x,Y}} \quad (14)$$

Due to the no-slip condition imposed upon viscous flows, heat transfer to or from the fluid at the wall may be expressed in terms of conduction through a thin film of stagnant fluid. Thus:

$$q'' = q/A = k \left(\frac{\partial t}{\partial y} \right)_{y=0} \quad (15)$$

which must equal the heat flux due to convection,

$$q'' = q/A = h(t_w - t_m) \quad (16)$$

Defining the local Nusselt number as;

$$Nu_x = \frac{hd}{k} \quad (17)$$

Combining (14) and (15),

$$Nu_x = \frac{\left(\frac{\partial t}{\partial y} \right)_{y=0}}{t_w - t_m} \quad (18)$$

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A Nusselt number based on the hydraulic diameter (d) instead of half the gap width (d) in finite difference form becomes;

$$N_{u_x} = \frac{4(T_w - T_{w-\Delta Y})}{\Delta Y} \frac{D}{T_w - T_m} \quad (19)$$

The hydraulic diameter was introduced in (19) to enable comparisons with previous solutions. The conversion is:

$$D = 4d$$

In addition, for the asymmetric constant wall temperature case, it is convenient to use a dimensionless parameter other than the Nusselt number. A dimensionless heat flux parameter may be defined by using (15) above:

$$\begin{aligned} q_f^* &= \frac{q D}{A k t_o} = \frac{D}{k t_o} \left[h \left(\frac{\partial t}{\partial y} \right)_{y=0} \right] \\ &= \frac{D}{k t_o} \left[\frac{k t_o}{d} \left(\frac{\partial T}{\partial Y} \right)_{Y=0} \right] \\ &= 4 \left(\frac{\partial T}{\partial Y} \right)_{Y=0} \end{aligned}$$

D. Thermal Boundary Conditions In Finite Difference Form

Constant Wall Temperature (Symmetric Case) Since the temperature distribution is symmetrical with respect to the centerline, computations were performed in only the upper half of the duct. A coordinate system was adapted with the X axis on the centerline of the channel. The thermal boundary condition at the wall was applied as:



$$t_w/t_o = T_w = N$$

where N is a positive integer.

Constant Heat Input (Symmetric Case) With the axis system as above, the slope of the thermal profile was maintained constant at the wall by adding a constant to the calculated temperature one step from the wall, thus:

$$T_w = T_w - \Delta Y + M$$

where: $M = (\Delta T / \Delta Y)_w \cdot \Delta Y$

Constant Wall Temperature with One Wall Insulated. For this case the coordinate system was redefined so the X axis coincided with the lower wall. The lower wall boundary condition was taken to be:

$$t_{w1}/t_o = T_{w1} = N$$

while zero slope at the upper wall was obtained by equating the wall temperature to the calculated temperature one step from the wall:

$$t_{w2}/t_o = T_{w2} = T_{w2} - \Delta Y$$

Constant Wall Temperature (Asymmetric Case). With the axis system the same as for the insulated wall case, thermal boundary conditions were established as;

$$t_{w1}/t_o = T_{w1} = N_1$$

$$t_{w2}/t_o = T_{w2} = N_2$$

where N_1 and N_2 are positive integers and $N_1 \neq N_2$.

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Noting that:

$$\frac{t_{w1} - t_o}{t_{w2} - t_o} = \frac{t_{w1}/t_o - 1}{t_{w2}/t_o - 1} = \frac{N_1 - 1}{N_2 - 1}$$

Values of $N_1 - 1/N_2 - 1$ of $1/2, 1/3, 1/4$ were investigated.

E. Computational Procedure

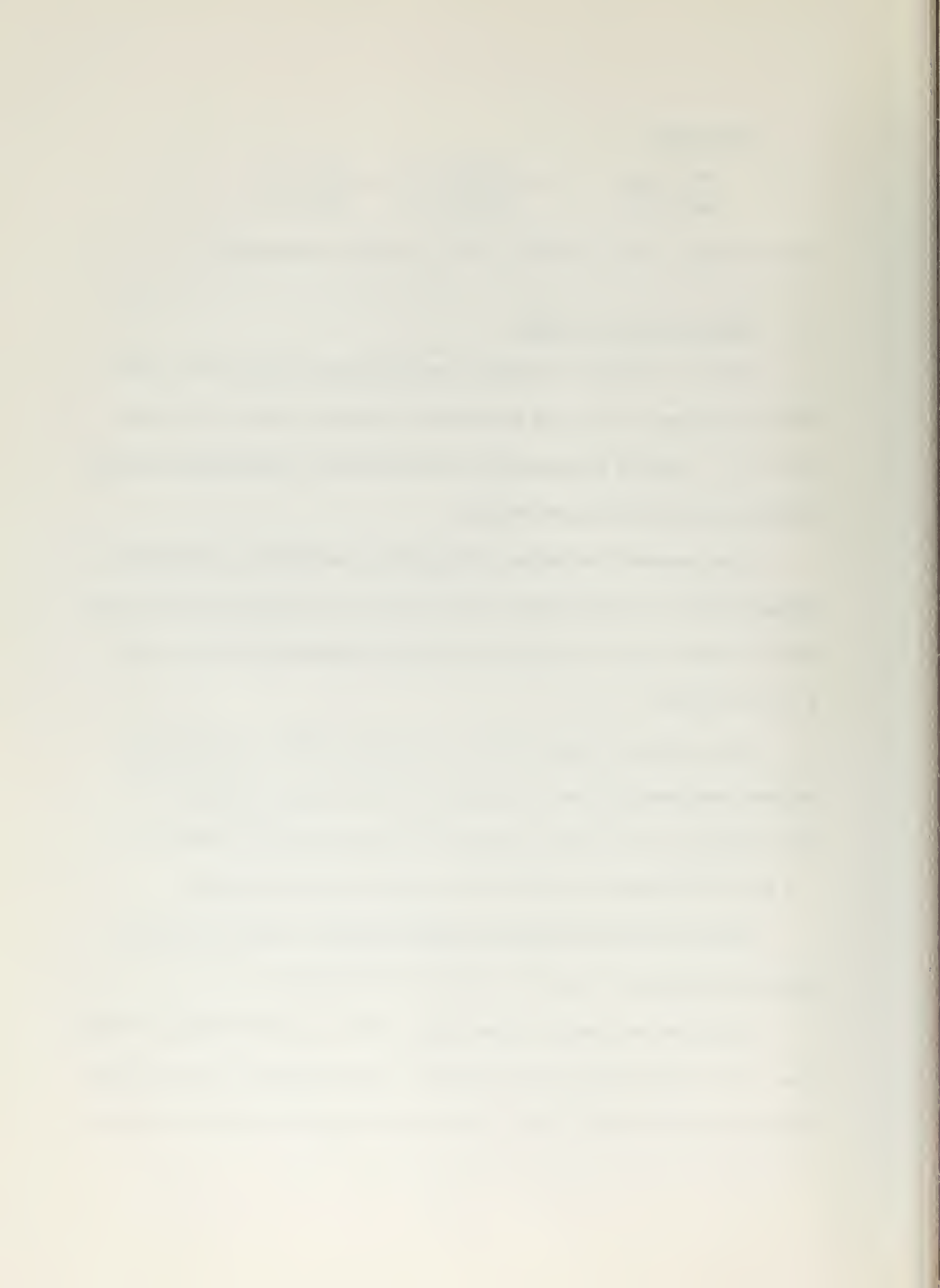
The finite difference solution was obtained by using a CDC 1604 digital computer to solve the equations at intervals of $\Delta X = 0.001$ and $\Delta Y = 0.1$. Velocity distributions were obtained by interpolating plotted curves of Bodoia's tabulated results.

Since the entire solution is dependent upon the first calculations at the entrance, starting values were obtained by reducing the grid size to $\Delta X = 0.0001$ and $\Delta Y = 0.05$ for the first 20 streamwise stations from $X = 0$ to 0.0020 .

The continuity equation (2) was evaluated and the cross-wise velocity component (V) was introduced into the energy equation (11) at each grid point. The energy solution was then obtained at intervals of Y between centerline and the wall at each streamwise station.

Solutions for each thermal boundary condition were obtained for Prandtl numbers of: $0.5, 0.7, 1.0, 1.6, 3.2$ and 10 .

Solutions for Prandtl numbers less than 0.5 are excluded by the grid size. This may be seen from inspection of the coefficient of the second temperature term in (11), which indicates the interdependence of $PR, \Delta X,$



and ΔY . In order to obtain convergence, PR must be large enough to prevent this term from becoming negative. Thus low Prandtl numbers, approaching those of liquid metals, cannot be effectively handled by finite-difference methods owing to the large number of calculations required by the small grid.

The computer program, written in Fortran 60, is included in the Appendix.

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III. RESULTS

Tables I through VI contain computed and extrapolated values of the heat transfer rates as a function of distance from the entrance for each of the thermal boundary conditions for a Prandtl number of 0.7. Figures 5 through 29 contain curves of the transfer rate versus downstream position and temperature and velocity profiles for Prandtl numbers of 0.5, 0.7, 1.0, 1.6, 3.2 and 10.

With the exception of the Asymmetric Constant Wall Temperature Case, which is expressed in terms of the dimensionless heat flux parameter, q^* ; the heat transfer rates are presented as local Nusselt number variation with the length parameter, L . Whereas, the computations were carried out using the dimensionless length parameter X , which is normally employed for hydrodynamic developments; L , based on the hydraulic diameter is customarily employed in heat transfer. The conversion is:

$$X = 16L$$

The use of all finite starting length" in the finite difference equations results in a finite initial value for the Nusselt number at $X = 0$ rather than the theoretically infinite value at the entrance. This causes a nearly flat slope near the entrance and an artificial inflection point in the Nusselt number versus length curve. The inflection point occurs progressively farther downstream with the increasing Prandtl number, thereby limiting the

CHAPTER I

The first part of the book is devoted to a general survey of the subject. It begins with a definition of the term "philosophy" and then proceeds to a discussion of the various branches of the subject. The author then discusses the history of philosophy, from the ancient Greeks to the modern era. He then discusses the various schools of thought, from the Stoics to the modern philosophers. The chapter concludes with a discussion of the importance of philosophy in the modern world.

The second part of the book is devoted to a detailed discussion of the various branches of philosophy. It begins with a discussion of metaphysics, which is the study of the nature of reality. The author then discusses epistemology, which is the study of knowledge. He then discusses ethics, which is the study of morality. The chapter concludes with a discussion of aesthetics, which is the study of art and beauty.

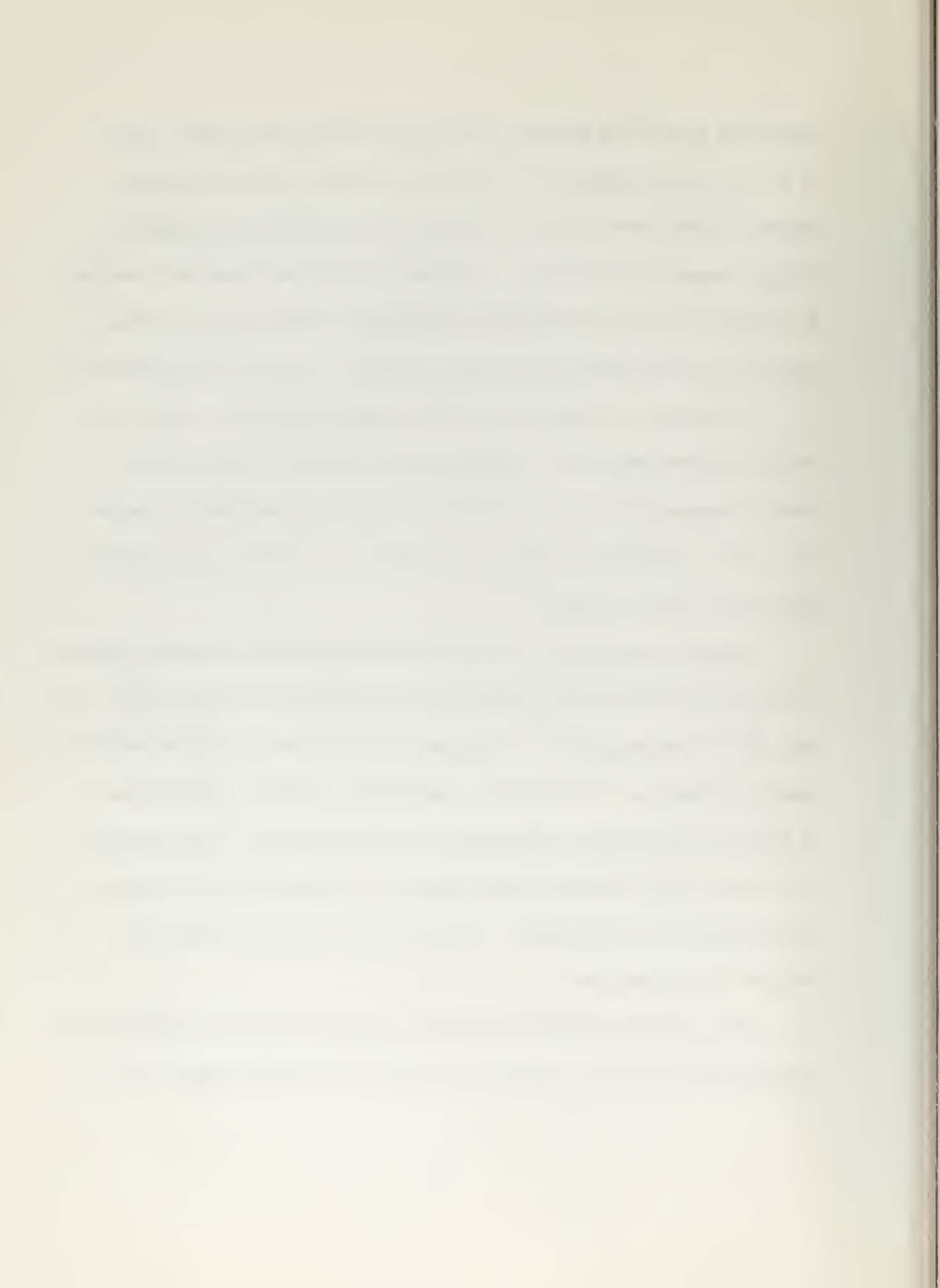
The third part of the book is devoted to a discussion of the various schools of thought. It begins with a discussion of the ancient Greeks, who were the first to develop a systematic philosophy. The author then discusses the medieval philosophers, who were influenced by the teachings of the Church. He then discusses the modern philosophers, who were influenced by the scientific revolution. The chapter concludes with a discussion of the various schools of thought in the modern world.

the useful span of the curves. A reduction of grid size from $\Delta X = .001$, $\Delta Y = .1$ to $\Delta X = .0001$, $\Delta Y = .05$ was observed to double the initial Nusselt number from 40 to 80, and move the inflection point upstream. This increased the useful span. However, the approach obviously reaches a practical limit very rapidly due to the number of calculations and the accuracy required of the hydrodynamic solution very close to the entrance.

In general, for the three thermal boundary conditions in which the Nusselt number was used as a heat transfer parameter, the curves for Prandtl numbers of 0.5, 0.7, 1.0 and 1.6 behave as expected for values of $L > 10^{-4}$. For Prandtl numbers of 3.2 and 10, $L = 10^{-3}$ is the approximate lower limit of validity.

Constant Heat Input. A comparison of local Nusselt numbers obtained in the present analysis with those reported by Siegel and Sparrow \overline{Nu}_7 , and Han \overline{Nu}_6 , contained in Fig. 5, indicate close agreement with the results of Siegel and Sparrow. Their solution employed a simplified energy equation in which the temperature distribution in the boundary layer was expressed as a series of polynomials in the transverse coordinate using the downstream station as a parameter. The hydrodynamic solution employed Schiller's approximation.

The comparison with Han's integro-numerical solution is not as close as might appear in Fig. 5 since this curve is for a Prandtl number of 0.8



and should lie above the other two curves which are for a Prandtl number of 0.7.

The computed curves of local Nusselt number versus length contained in Fig. 6. for Prandtl numbers of 0.5, 0.7, 1.0, 1.6, 3.2, and 10, may be approximated within 10 per cent, for $0.5 \leq PR \leq 1.6$, and $L > 10^{-3}$ by the following;

$$Nu_x = 8.24 + \frac{.0186 (PR/L)}{1 + .0178 (PR/L)^{.6}}$$

Velocity and temperature profiles for constant heat input are contained in Figs. 7 through 10.

Constant Wall Temperature (Symmetric Case). Local values of Nusselt numbers for this boundary condition are compared in Fig. 11 with those of Sparrow $\overline{[5]}$, which were computed using the Karman-Pohlhausen method and Schiller's hydrodynamic solution. Again close correlation is indicated between the two results.

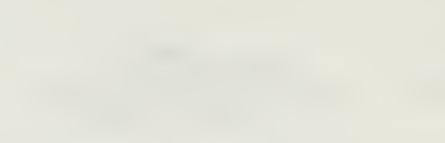
The local Nusselt number curves of Fig. 12 may be approximately represented within 10 per cent for $0.5 \leq PR \leq 1.6$, and $L > 10^{-3}$ by the following;

$$Nu_x = 7.60 + \frac{.0148 (PR/L)}{1 + .0143 (PR/L)^{.6}}$$

Velocity and temperature profiles for this boundary condition are contained in Figs. 13 through 16.

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One Wall Constant Temperature - One Wall Insulated. The results of this computation are presented in Fig. 17. The local Nusselt number may be approximated within 10 per cent for $0.5 \leq PR \leq 1.6$, and $L > 10^{-3}$ by;

$$N_{u_x} = 4.84 + \frac{.0155 (PR/L)}{1 + .012 (PR/L)^{.6}}$$

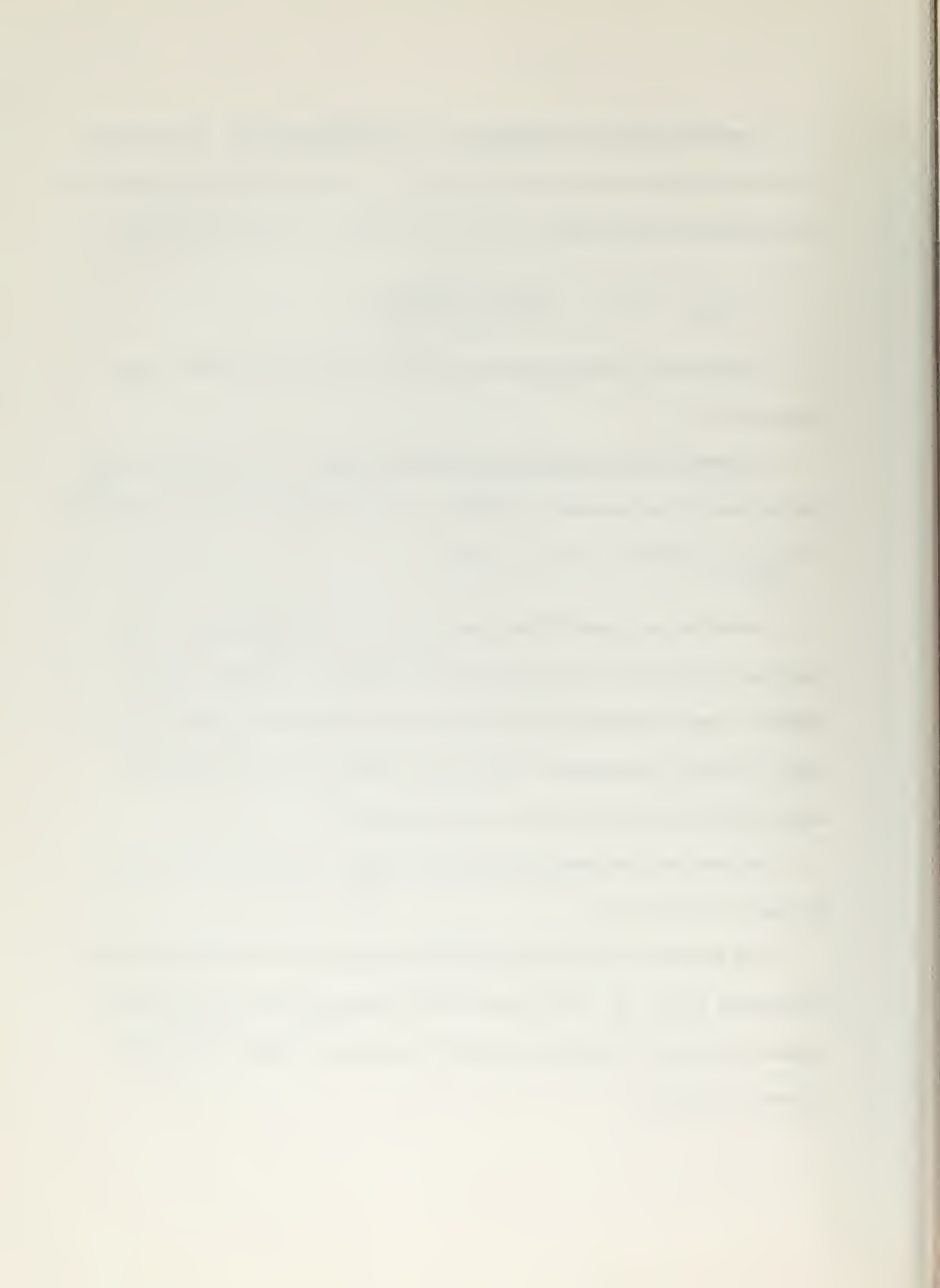
The velocity and temperature distributions are illustrated in Figs. 18 through 21.

Constant Wall Temperature (Asymmetric Case). Dimensionless heat flux values for this boundary condition are presented in Figs. 22, 23 and 24 for $\frac{N_1 - 1}{N_2 - 1}$ ratios of $1/2$, $1/3$, and $1/4$.

Note that the upper (hotter) wall curve is smoothly asymptotic to the value predicted for the steady state condition. The lower curve exhibits a much shallower initial slope, passes through an inflection point, and then, approaches a negative asymptotic value of the same magnitude and at the same rate as the upper wave.

Velocity and temperature profiles for $\frac{N_1 - 1}{N_2 - 1} = 1/4$ are presented in Figs. 25 through 28.

The behavior of the local Nusselt number for this asymmetrical case is depicted in Fig. 29. The lower curve, representing the local Nusselt number variation for the lower (cooler) wall, does not depict the actual physical situation.



Both the denominator and the numerator of;

$$N_{u_x} = \frac{\frac{4 (T_w - T_{w-\Delta Y})}{\Delta Y}}{T_w - T_m} \quad (18)$$

approach zero at different rates. The curve tends to positive infinity as the denominator approaches zero, returns from negative infinity, and becomes zero as the numerator goes to zero; and finally approaches the actual steady asymptotic value.



IV. CONCLUSIONS

From the results of this study it may be concluded that:

1. The finite difference method, as employed in this analysis, yields results which are consistent with previous solutions for other geometries $\sqrt{2}$ and which are asymptotic to the known fully developed values. The method is, however, limited to a small range of Prandtl numbers near unity by the small grid sizes necessary outside this range. For small Prandtl numbers the grid must be very small to obtain convergence, while for large Prandtl numbers practical grid dimensions result in significant inaccuracies near the entrance due to the "finite starting length".
2. The results of the approximate integral methods suggested by Siegel and Sparrow $\sqrt{7}$ for constant heat input, and by Sparrow $\sqrt{5}$ for constant wall temperature, compare very favorably with the finite difference solutions reported here. This additional substantiation of the methods suggested by these authors is significant because of the practical utility of their approach, which requires relatively few calculations, is capable of handling a wide range of Prandtl numbers, and can predict heat transfer rates very close to the entrance.

CHAPTER IV

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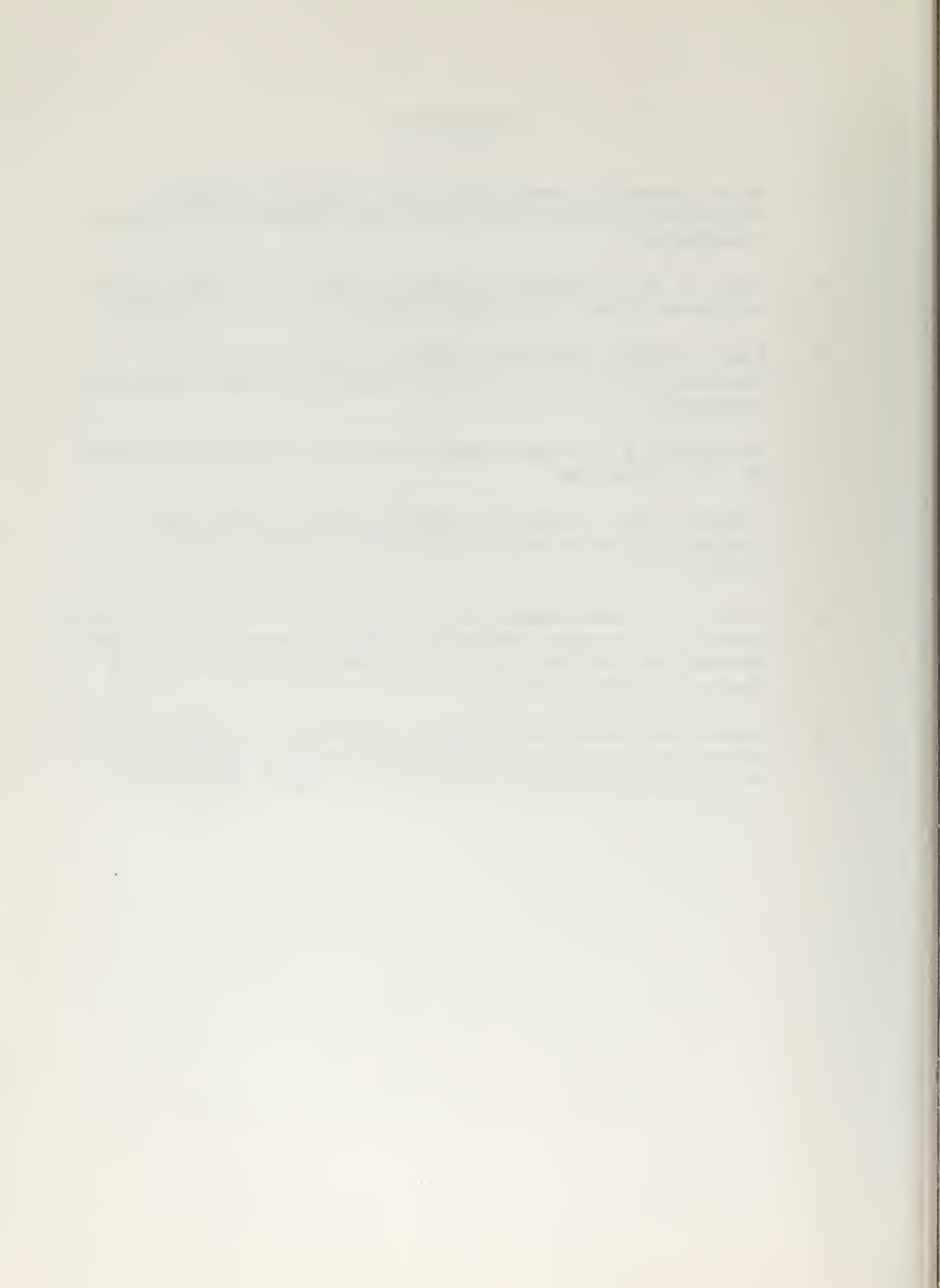


TABLE I
 CONSTANT HEAT INPUT
 $PR = 0.7$

X	L	N_{u_x}	T_w
.0016	.0001	46.2	1.09
.0032	.0002	30.0	1.14
.0064	.0004	23.7	1.18
.0128	.0008	18.0	1.24
.0160	.0010	16.6	1.26
.0320	.0020	12.9	1.35
.0640	.0040	10.3	1.47
.1280	.0080	8.87	1.62
.1600	.0100	8.62	1.67
Asymptotic to		8.24	



TABLE II
 CONSTANT WALL TEMPERATURE
 (SYMMETRIC CASE)

$PR = 0.7$

X	L	N_{u_x}
.0016	.0001	34.0
.0032	.0002	26.0
.0064	.0004	19.9
.0128	.0008	15.3
.0160	.0010	13.7
.0320	.0020	10.5
.0640	.0040	8.58
.1280	.0080	7.78
.1600	.0010	7.63
Asymptotic to		7.60



TABLE III

ONE WALL CONSTANT TEMPERATURE - ONE WALL INSULATED

 $PR = 0.7$

X	L	N_{u_x}	T_w
.0016	.0001	33.0	1.000
.0032	.0002	24.0	1.000
.0064	.0004	19.2	1.000
.0128	.0008	13.0	1.000
.0160	.0010	11.6	1.000
.0320	.0020	9.05	1.000
.0640	.0040	7.20	1.000
.1280	.0080	5.93	1.001
.1600	.0100	5.55	1.021
.4000	.0250	4.96	1.139
Asymptotic to		4.84	2.000

TABLE IV
CONSTANT WALL TEMPERATURE
(ASYMMETRIC CASE)

$$\frac{N_1 - 1}{N_2 - 1} = 1/2 \quad PR = 0.7$$

X	L	q_1^*	q_2^*
.0032	.0002	26.2	57.5
.0064	.0004	17.3	41.4
.0128	.0008	11.9	28.7
.0160	.0010	10.7	25.9
.0320	.0020	8.08	18.7
.0460	.0040	6.18	13.7
.1280	.0080	4.73	10.1
.1600	.0100	4.24	9.15
.3200	.0200	2.24	6.35
.6400	.0400	-.19	3.81
.9600	.0600	-1.23	2.76
	Asymptotic to	-2.00	+2.00

TABLE V
CONSTANT WALL TEMPERATURE
(ASYMMETRIC CASE)

$$\frac{N_1 - 1}{N_2 - 1} = 1/3 \quad PR = 0.7$$

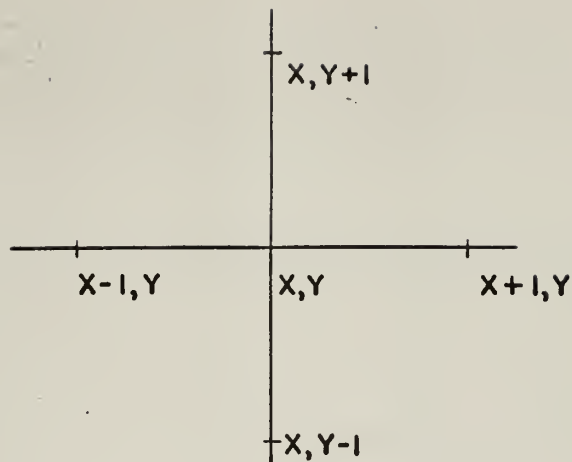
X	L	q_1^*	q_2^*
.0032	.0002	28.0	86.0
.0064	.0004	17.3	62.0
.0128	.0008	11.8	43.0
.0160	.0010	10.8	38.9
.0320	.0020	8.09	28.1
.0640	.0040	6.18	20.6
.1280	.0080	4.69	15.2
.1600	.0100	4.15	13.8
.3200	.0200	1.65	9.84
.6400	.0400	-1.58	6.41
.9600	.0600	-2.97	5.02
	Asymptotic to	-4.00	+4.00



TABLE VI
CONSTANT WALL TEMPERATURE
(ASYMMETRIC CASE)

$\frac{N_1 - 1}{N_2 - 1} = 1/4$		PR = 0.7	
X	L	q_1^*	q_2^*
.0032	.0002	28.2	116.0
.0064	.0004	16.8	82.9
.0128	.0008	12.2	57.7
.0160	.0010	10.5	50.4
.0320	.0020	8.09	37.5
.0640	.0040	6.17	27.5
.1280	.0080	4.66	20.3
.1600	.0100	4.06	18.4
.3200	.0200	1.06	13.3
.6400	.0400	-2.96	9.03
.9600	.0600	-4.71	7.27
	Asymptotic to	-6.00	+6.00





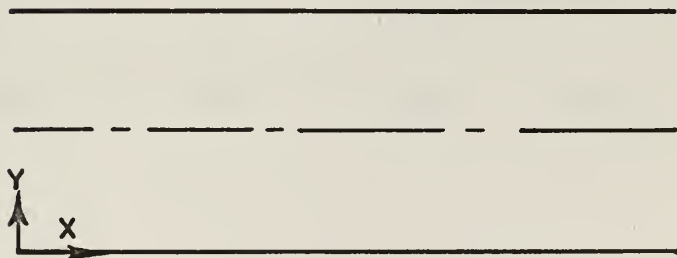
(a)

DESIGNATION OF MESH POINTS



(b)

AXIS SYSTEM, SYMMETRICAL CASES;
CONSTANT WALL TEMPERATURE
CONSTANT HEAT INPUT



(c)

AXIS SYSTEM, NON-SYMMETRICAL CASES;
ONE WALL INSULATED;
CONSTANT, BUT DIFFERENT WALL TEMPERATURES

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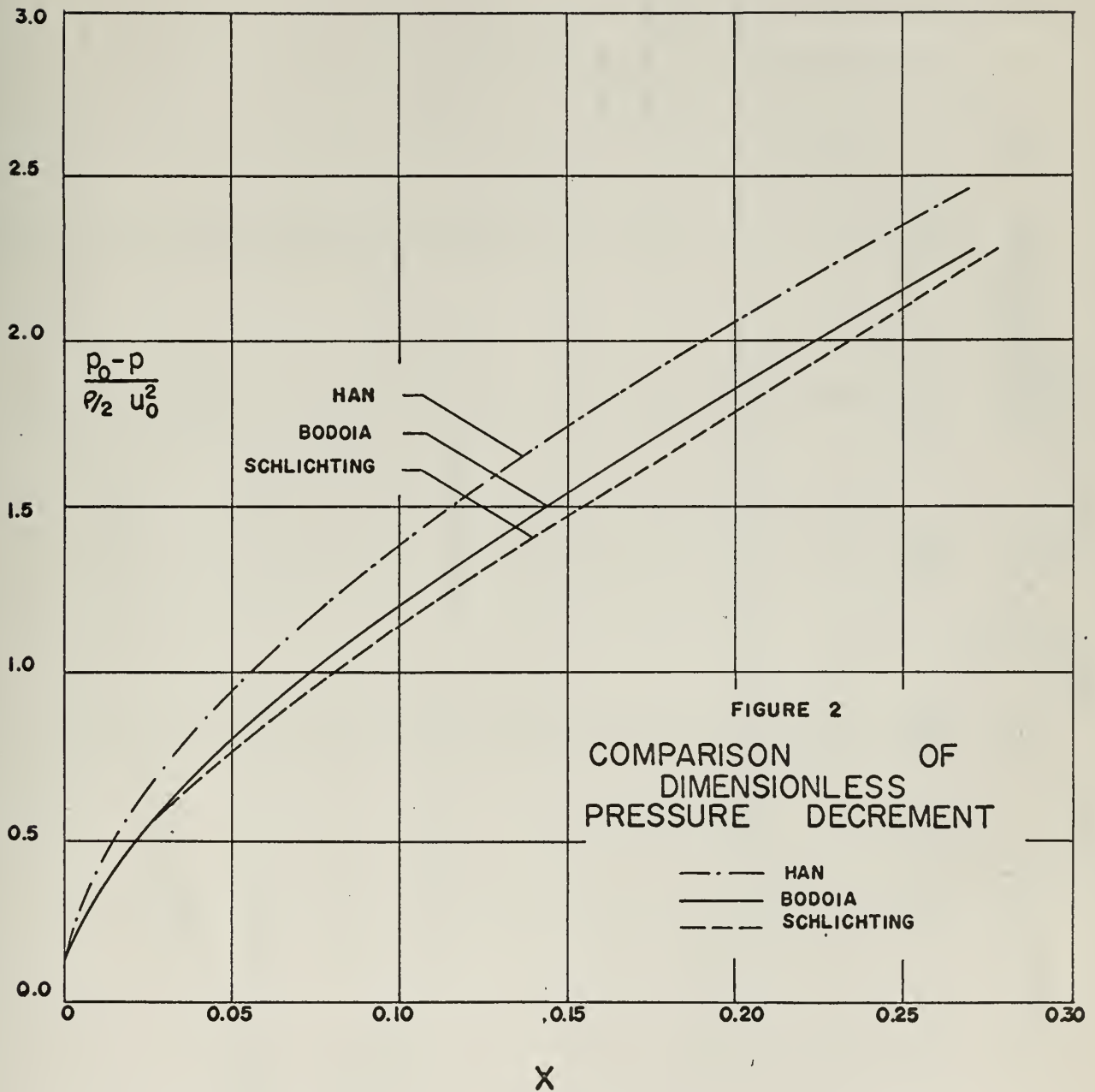
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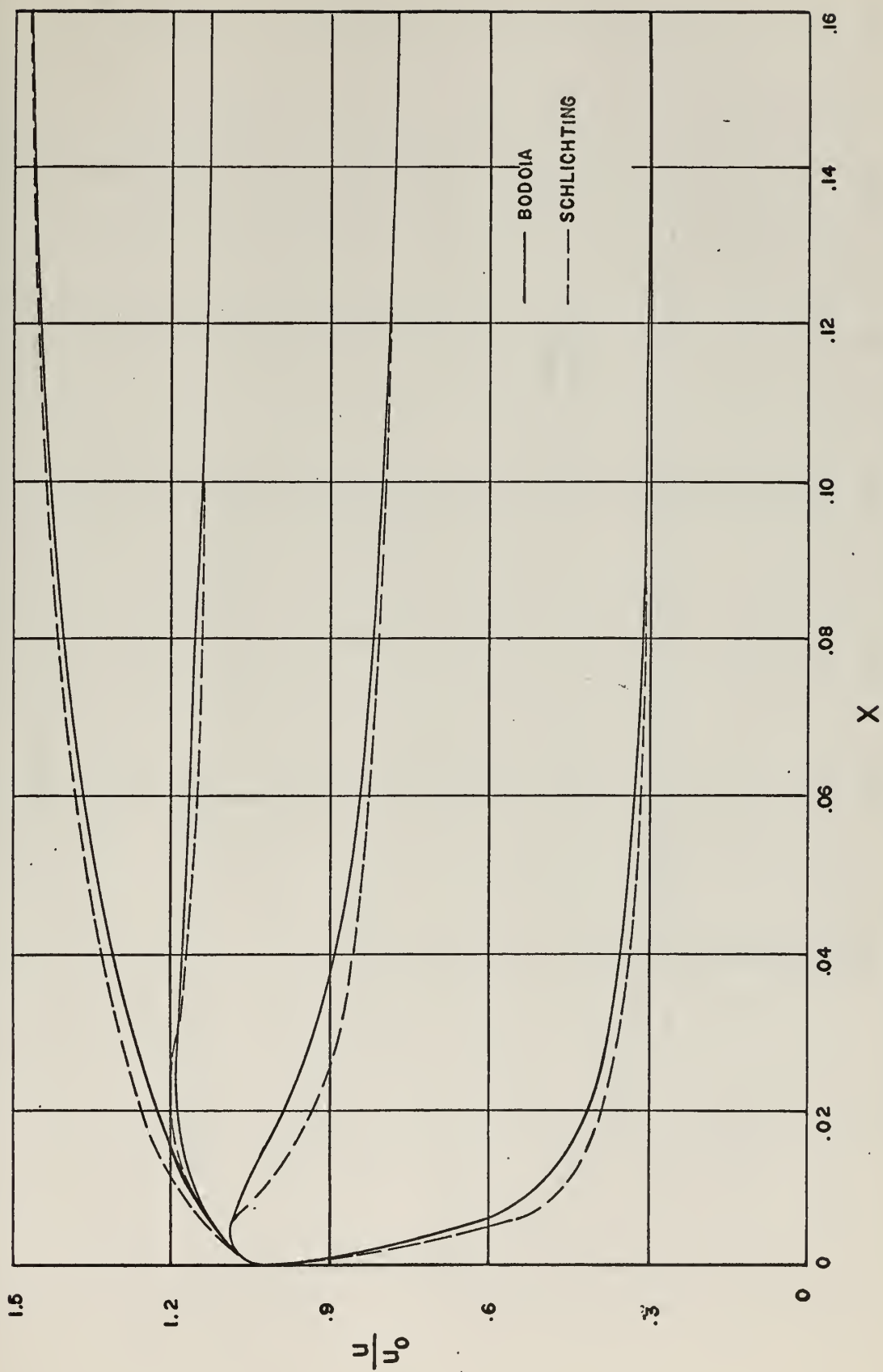
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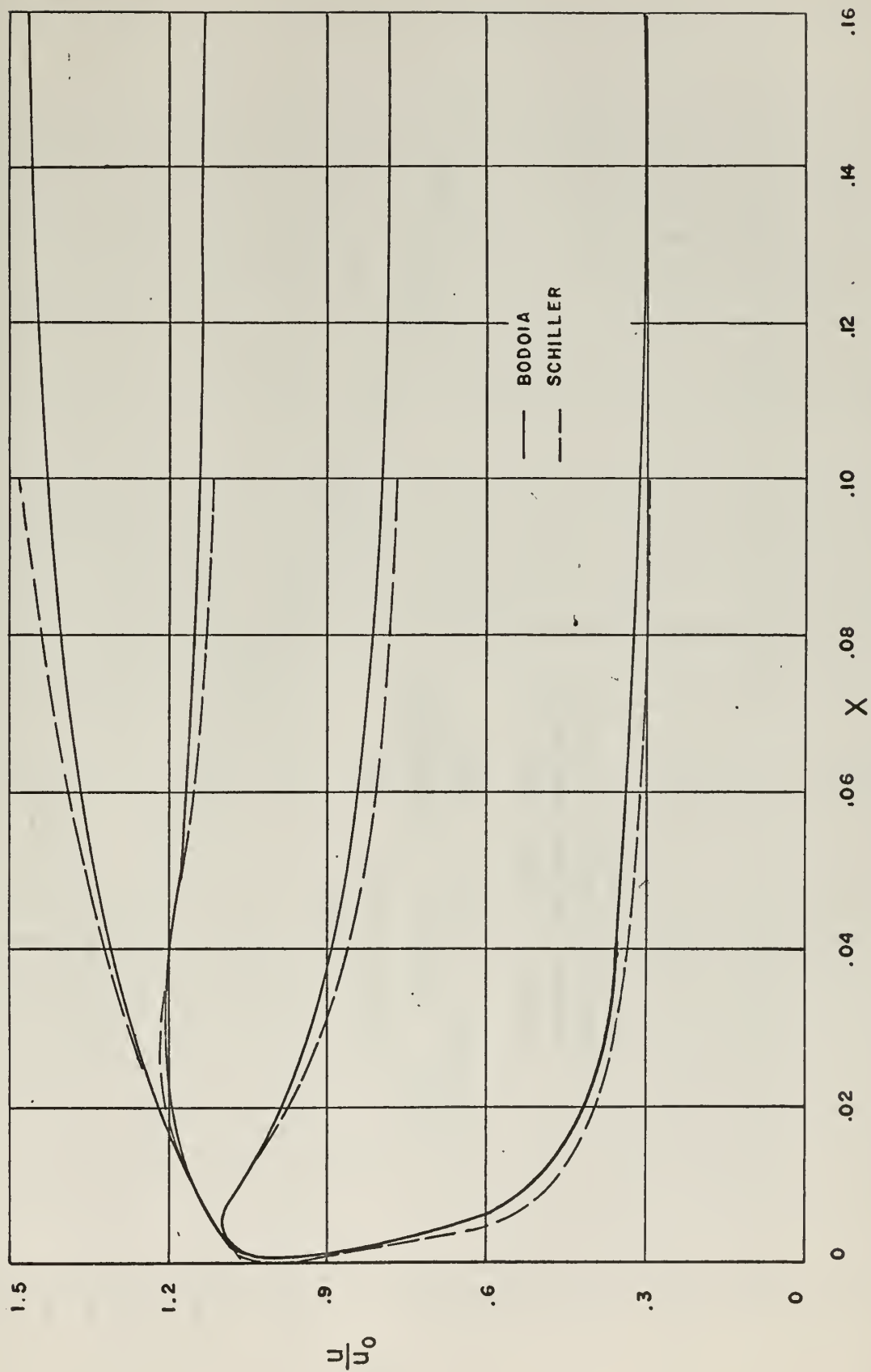
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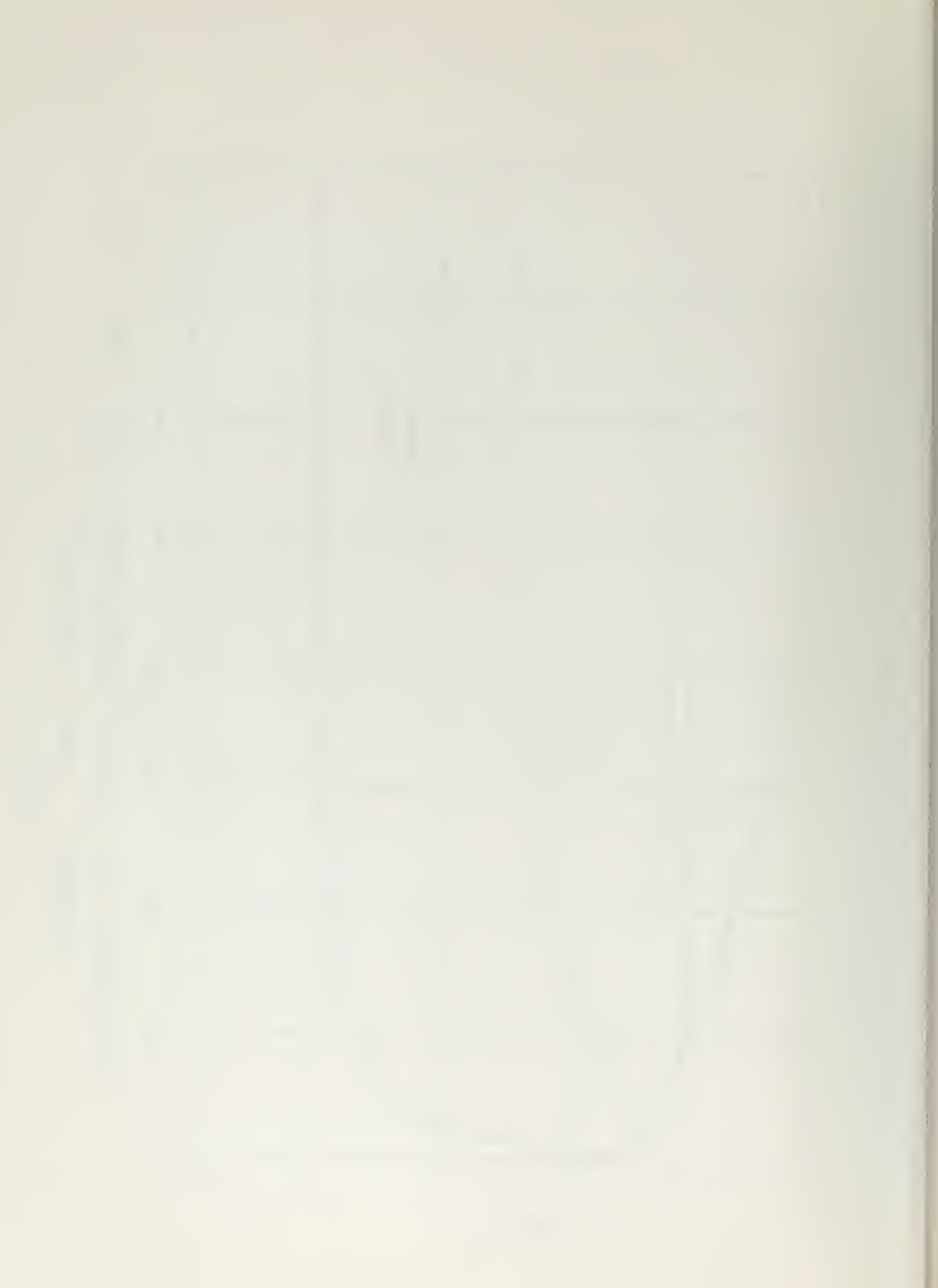


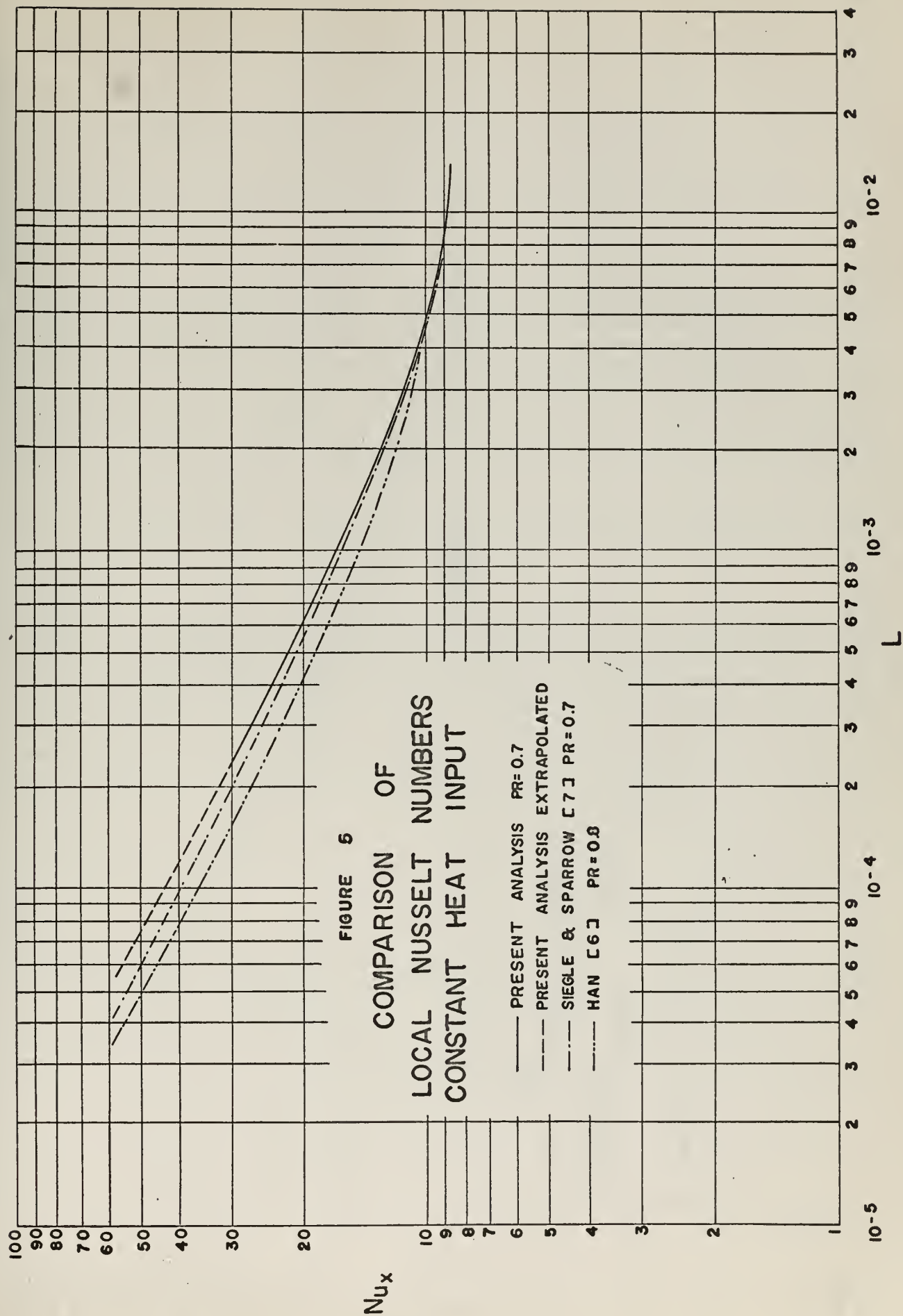
COMPARISON OF VELOCITY DISTRIBUTIONS OF
BODOIA AND SCHLICHTING



COMPARISON OF VELOCITY DISTRIBUTIONS DUE TO
BODOIA AND SCHILLER

FIGURE 4







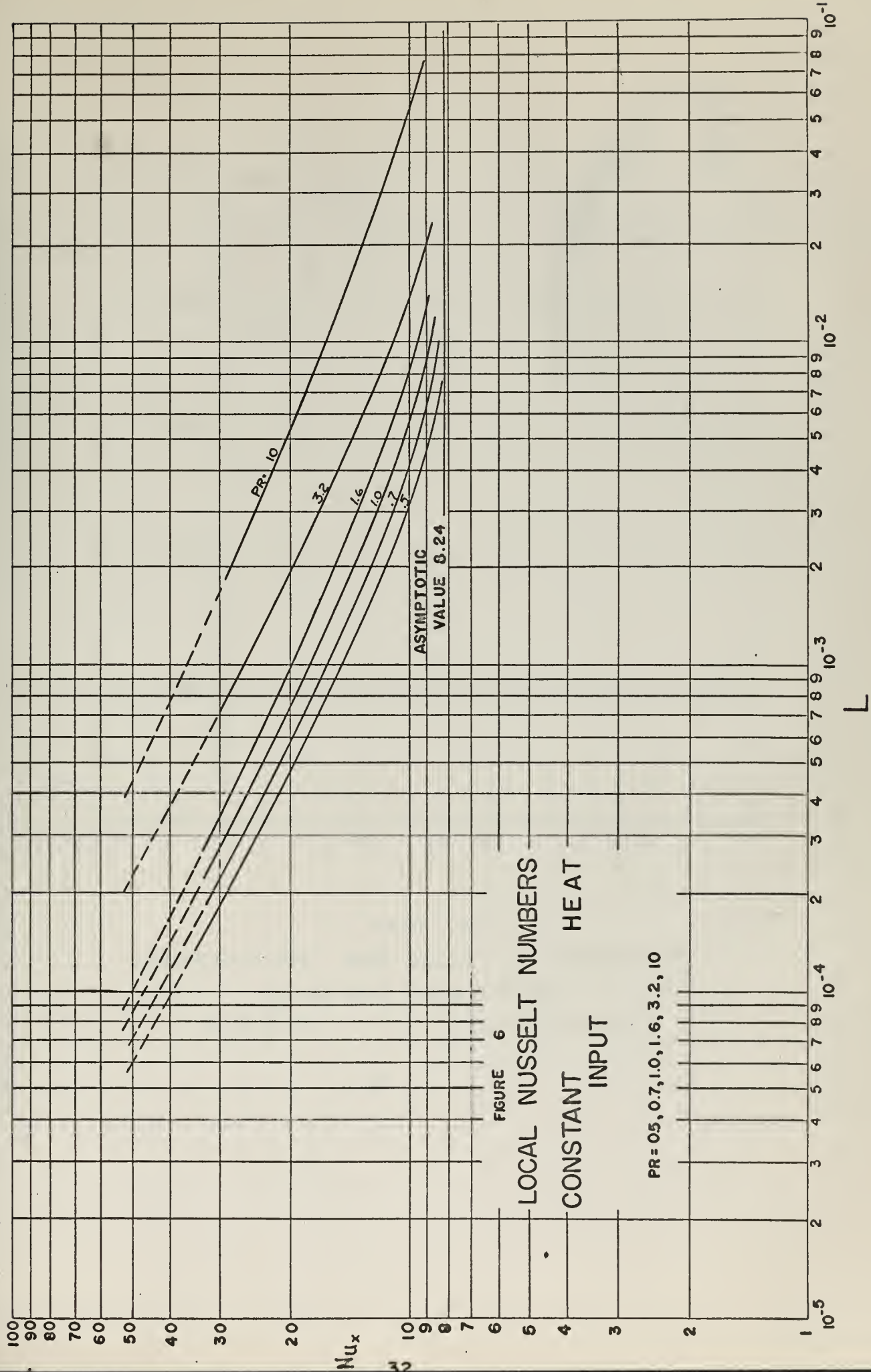


FIGURE 6
 LOCAL NUSSELT NUMBERS
 CONSTANT HEAT INPUT

$Pr = 0.5, 0.7, 1.0, 1.6, 3.2, 10$



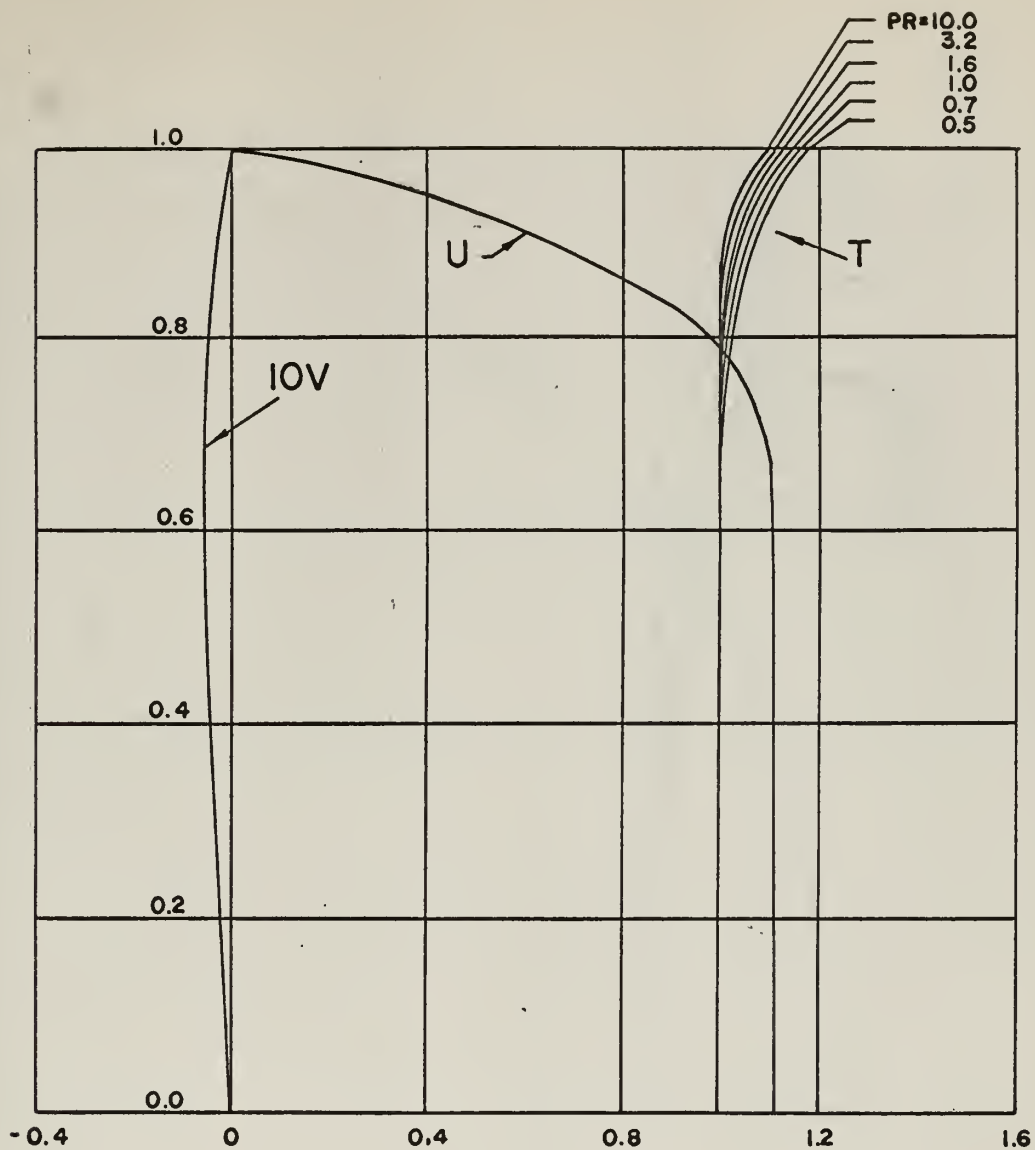


FIGURE 7
 TEMPERATURE AND VELOCITY PROFILES
 CONSTANT HEAT INPUT
 $X=0.005$ $L=0.0003125$



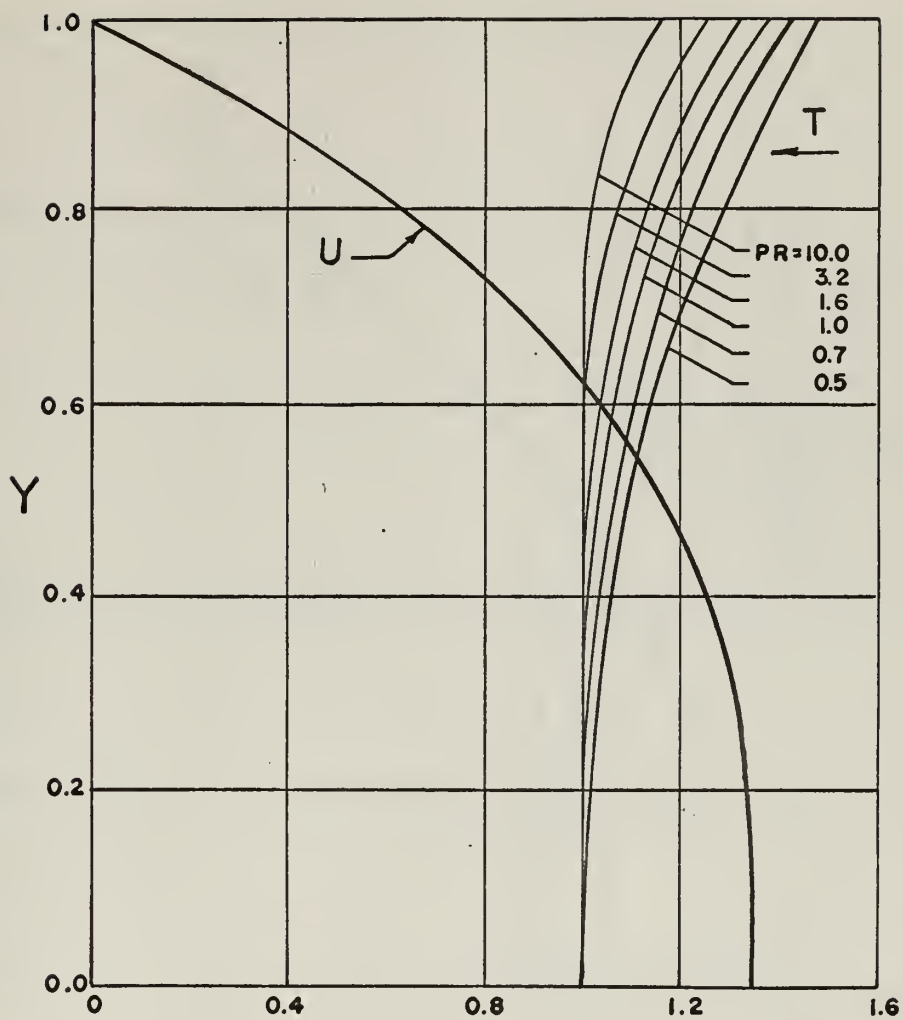


FIGURE 8

TEMPERATURE AND VELOCITY PROFILES
 $X = 0.050$ $L = 0.003125$



Figure 1

Figure 1 shows the concentration of the substance over time. The concentration increases rapidly and then levels off, approaching a horizontal asymptote at approximately 10 mg/L.

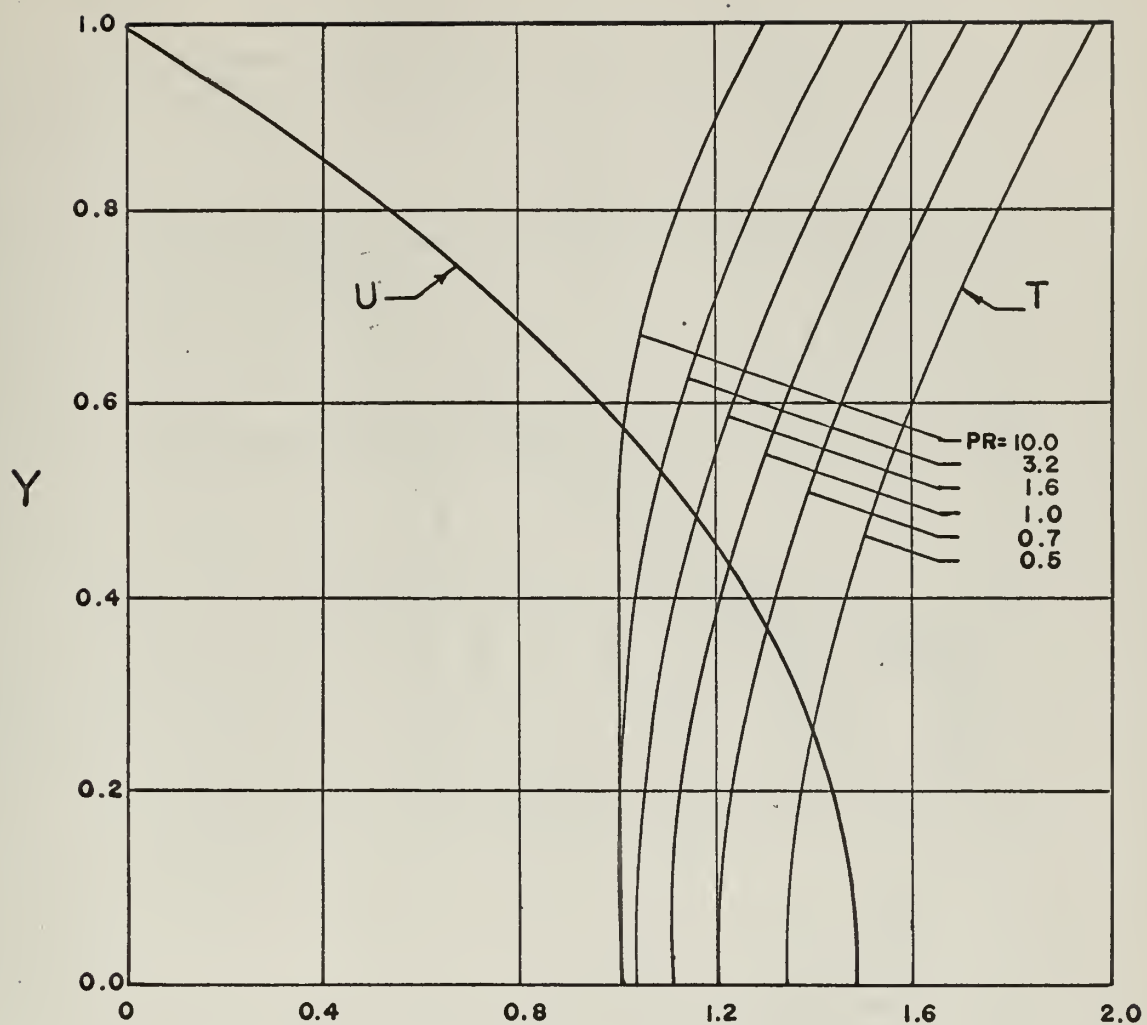


FIGURE 9

TEMPERATURE AND VELOCITY PROFILES

$X = 0.250$

$L = 0.0156$



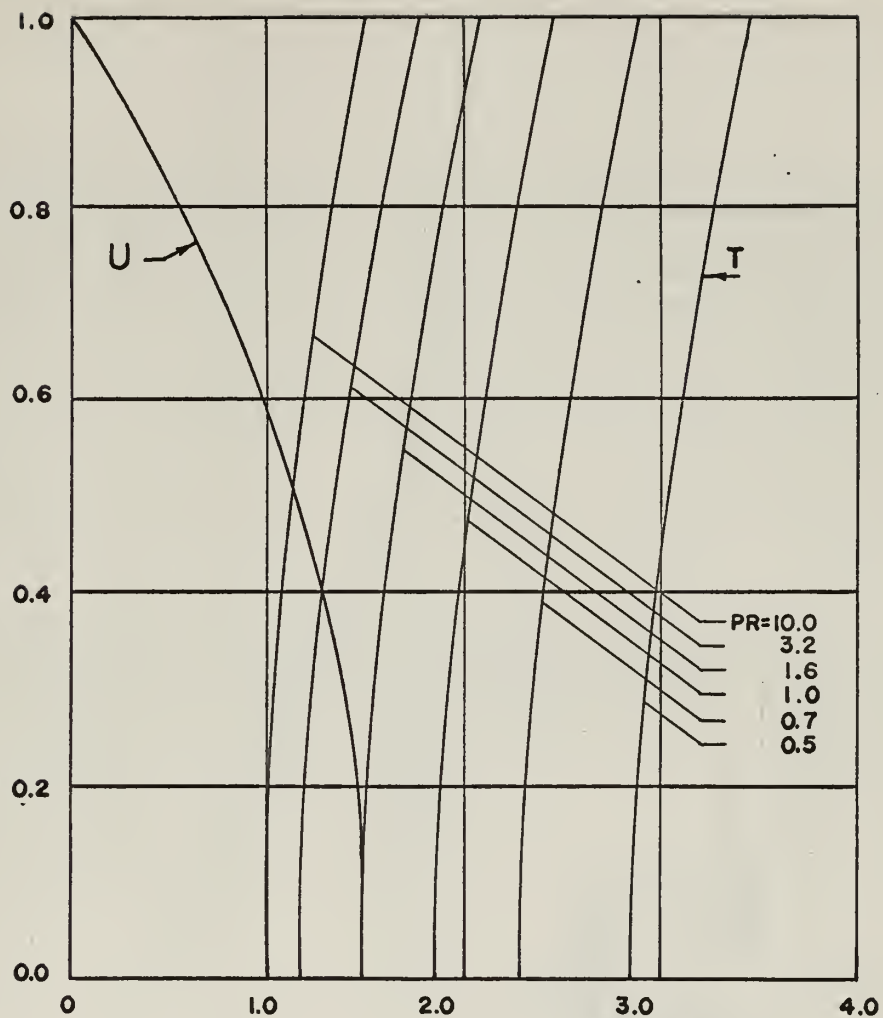


FIGURE 10

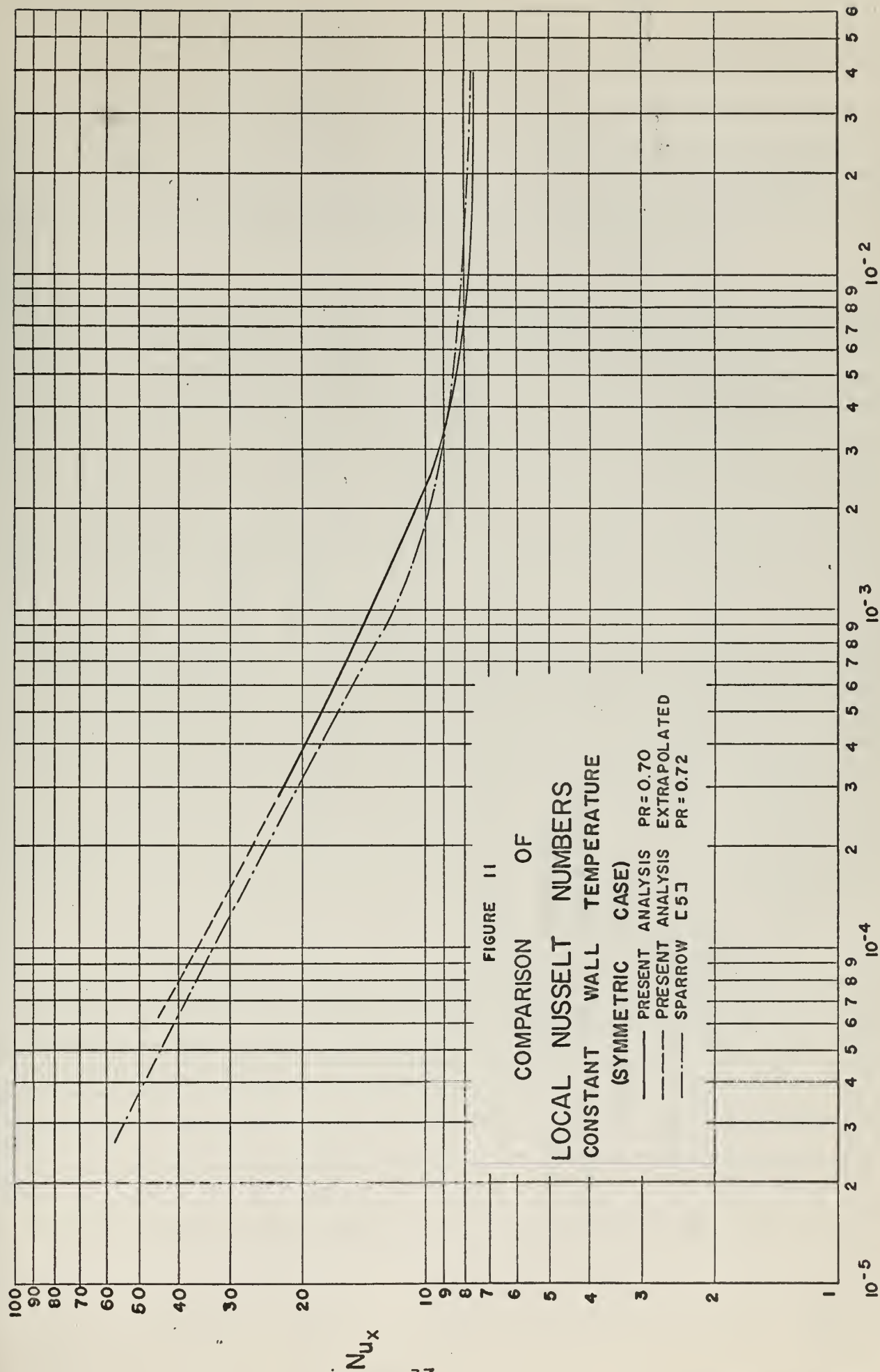
TEMPERATURE AND VELOCITY PROFILES

$X = 1.00$

$L = 0.0625$



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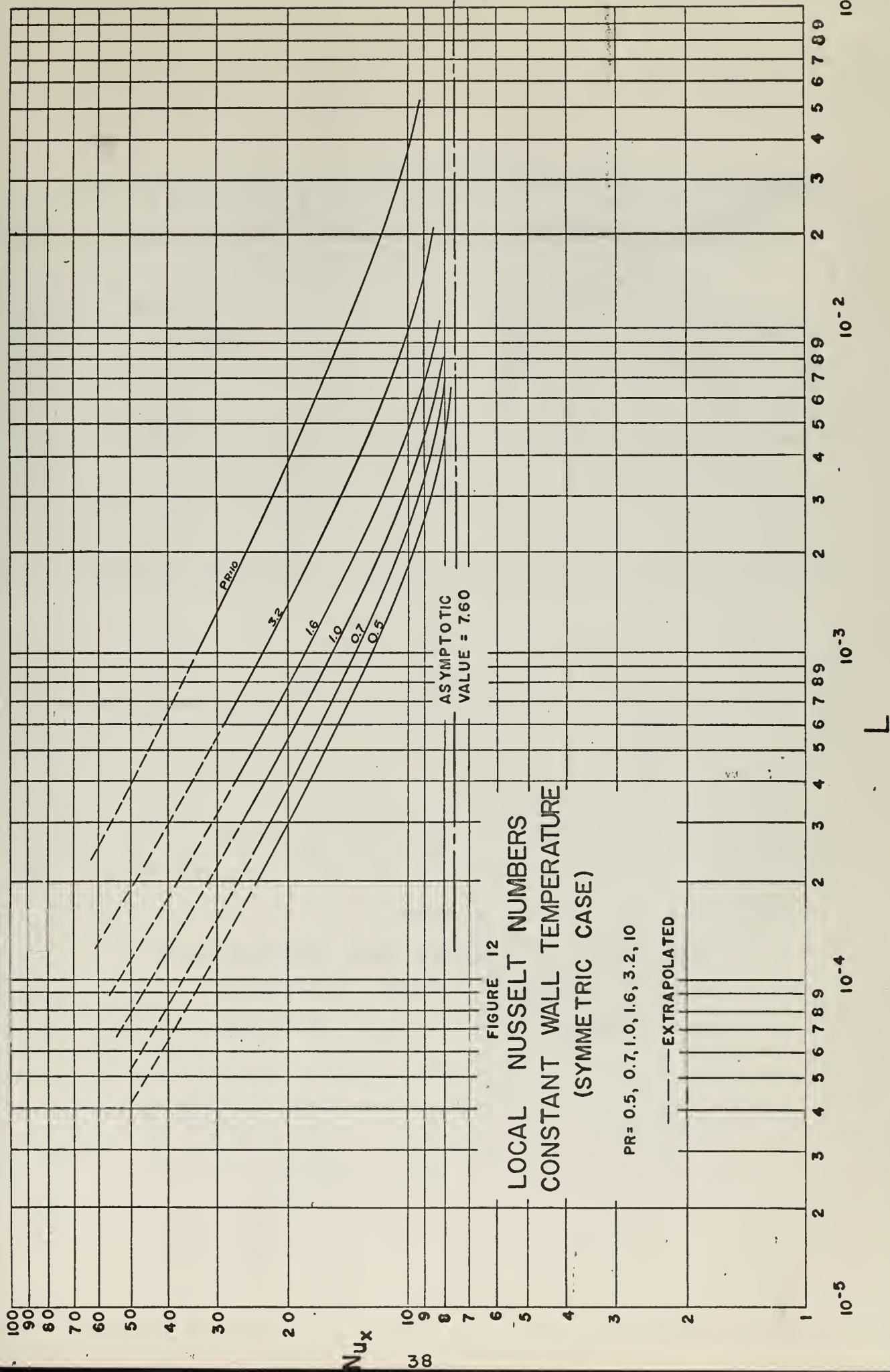
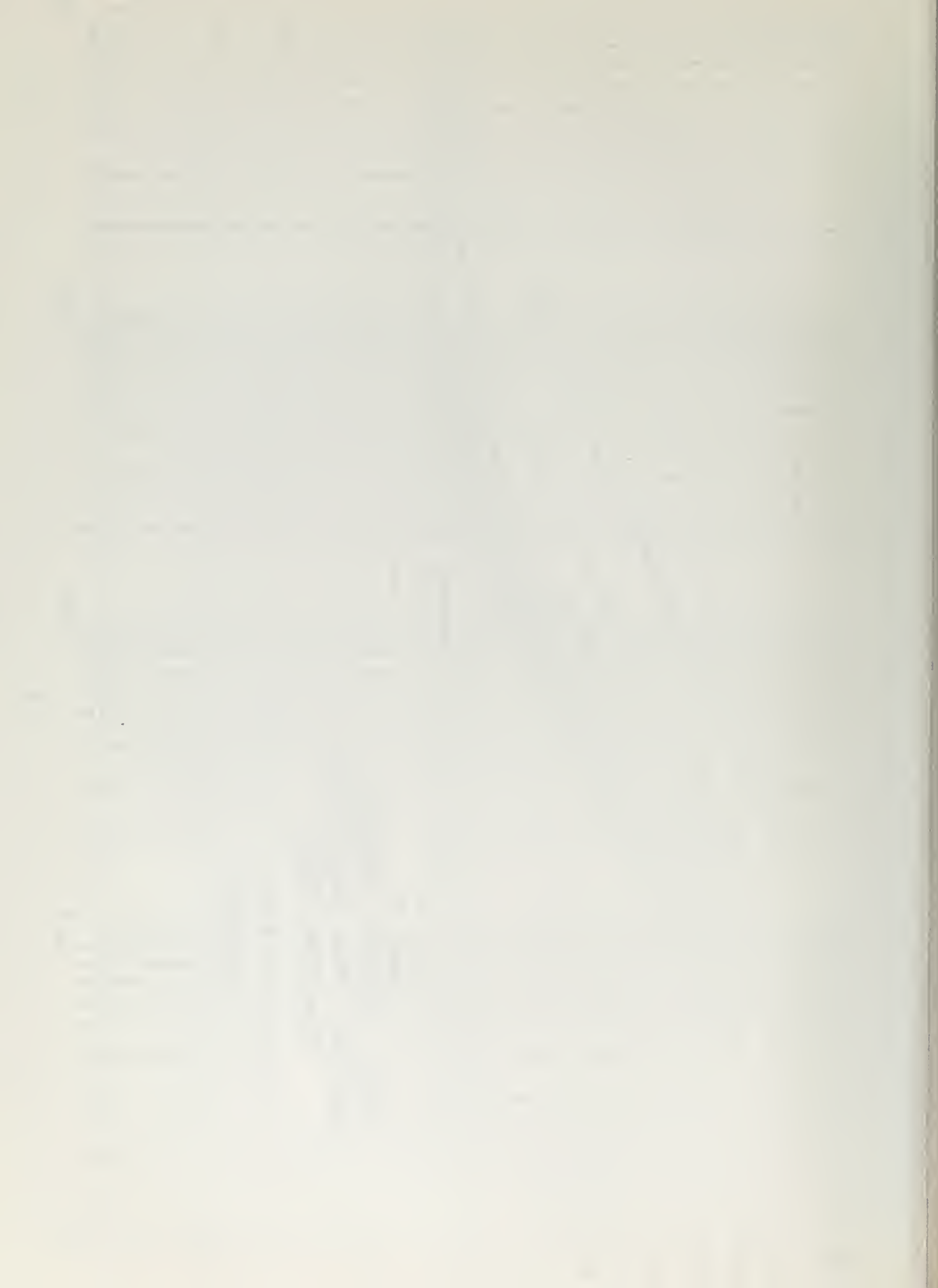


FIGURE 12
 LOCAL NUSSOLT NUMBERS
 CONSTANT WALL TEMPERATURE
 (SYMMETRIC CASE)

$Pr = 0.5, 0.7, 1.0, 1.6, 3.2, 10$

--- EXTRAPOLATED

ASYMPTOTIC
 VALUE = 7.60



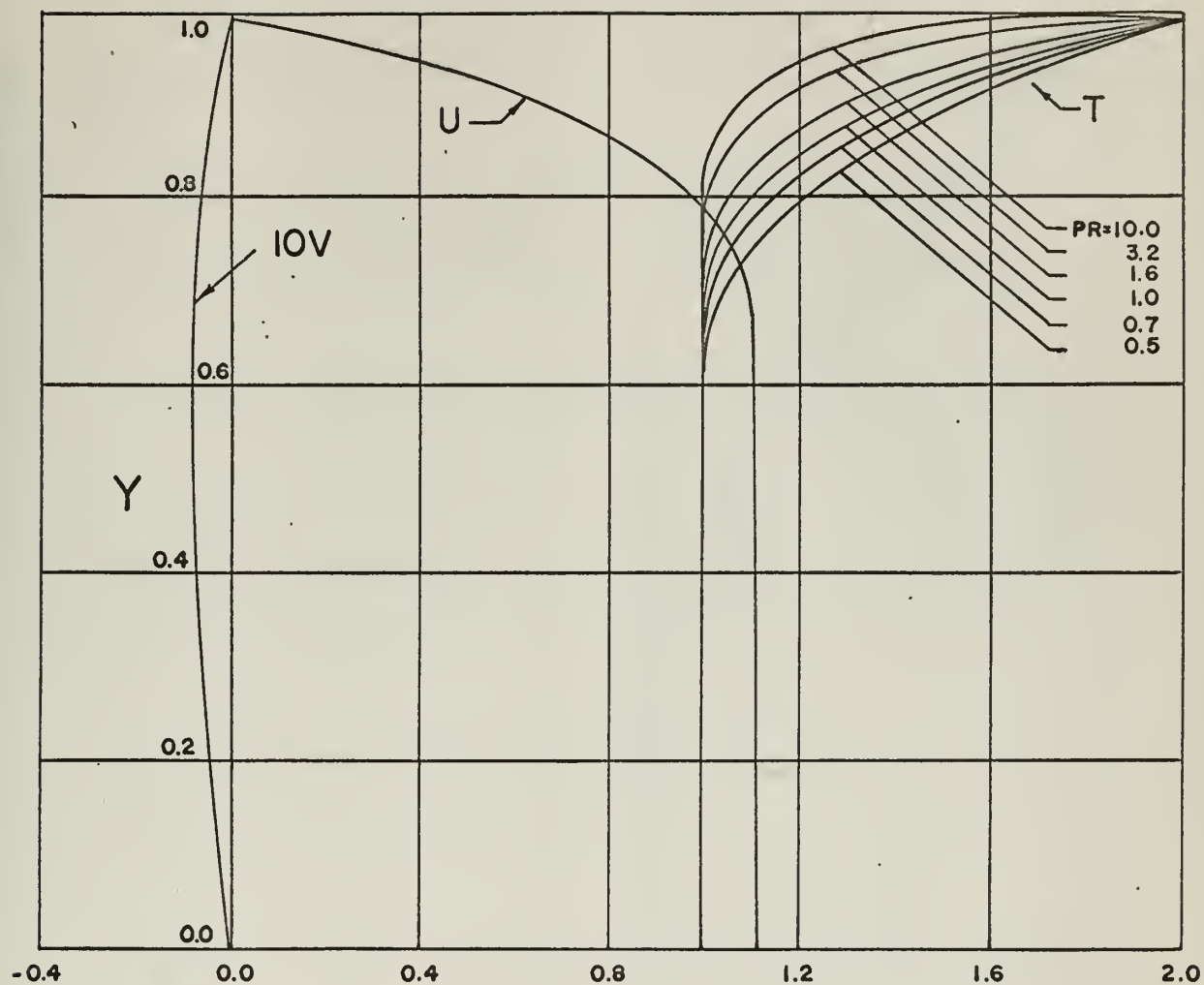


FIGURE 13

TEMPERATURE AND VELOCITY PROFILES
 CONSTANT WALL TEMPERATURE
 $X=0.005$ $L = 0.0003125$



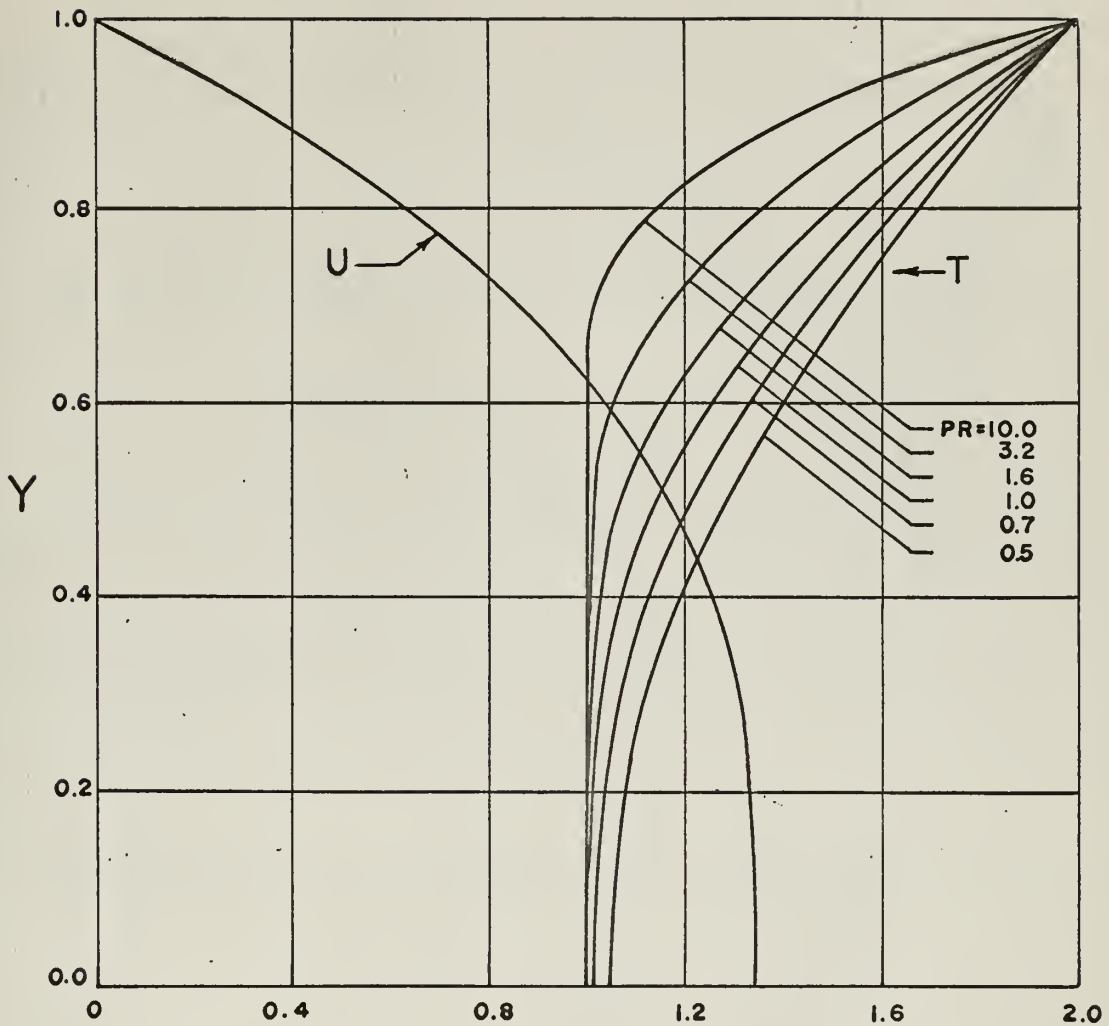


FIGURE 14

TEMPERATURE AND VELOCITY PROFILES
 CONSTANT WALL TEMPERATURE
 $X = 0.050$ $L = 0.003125$



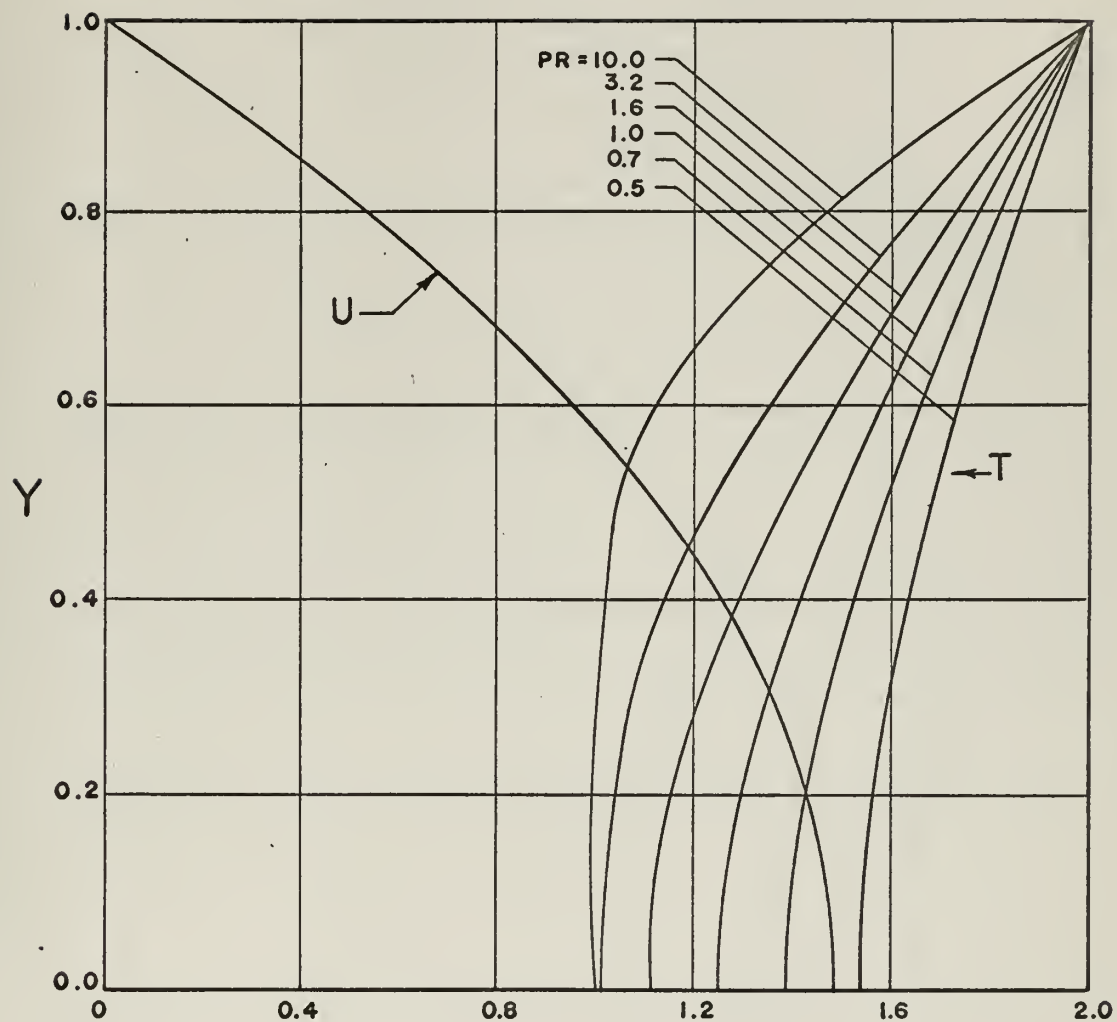


FIGURE 15

TEMPERATURE AND VELOCITY PROFILES
 CONSTANT WALL TEMPERATURE
 $X = 0.250$ $L = 0.0156$



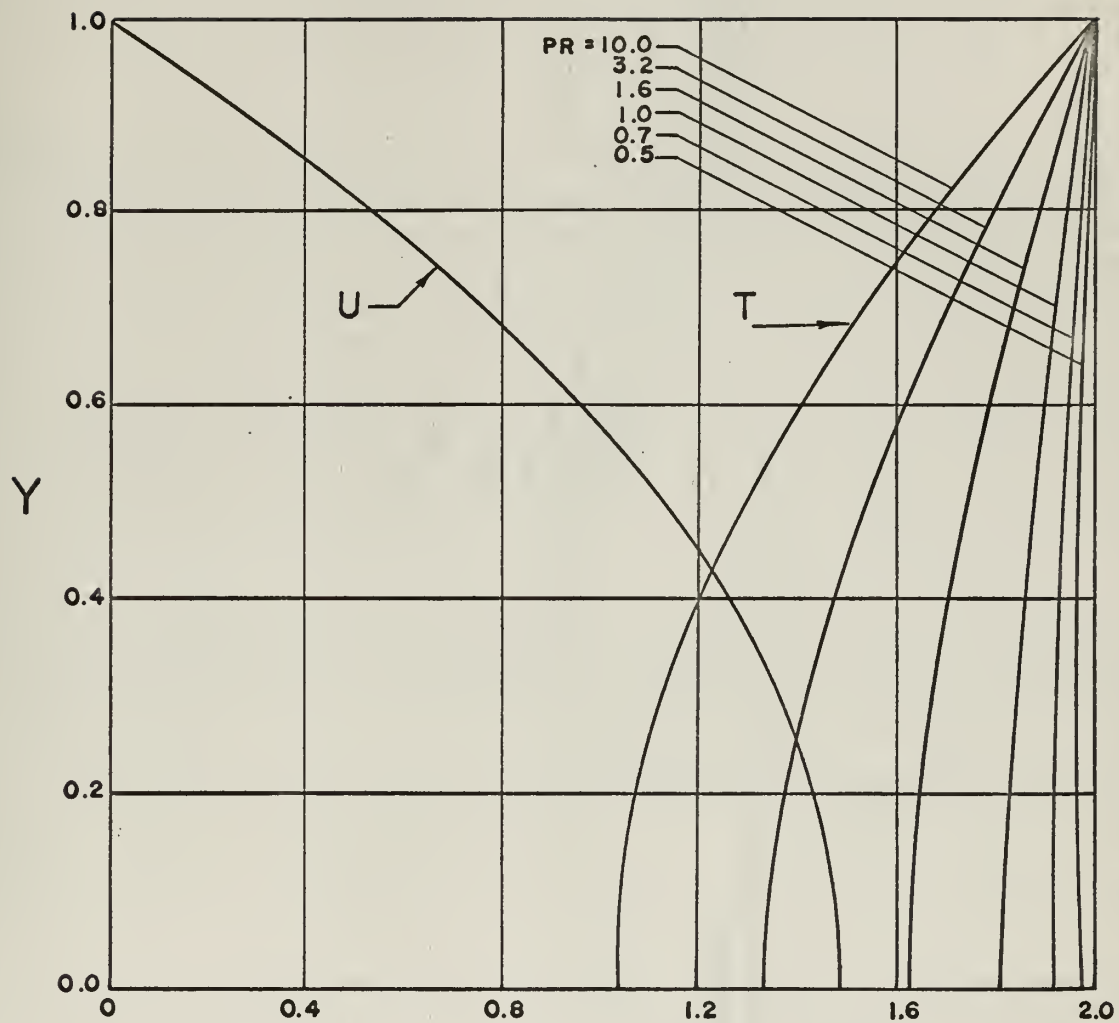


FIGURE 16

TEMPERATURE AND VELOCITY PROFILES
 CONSTANT WALL TEMPERATURE
 $X = 1.00$ $L = 0.0625$



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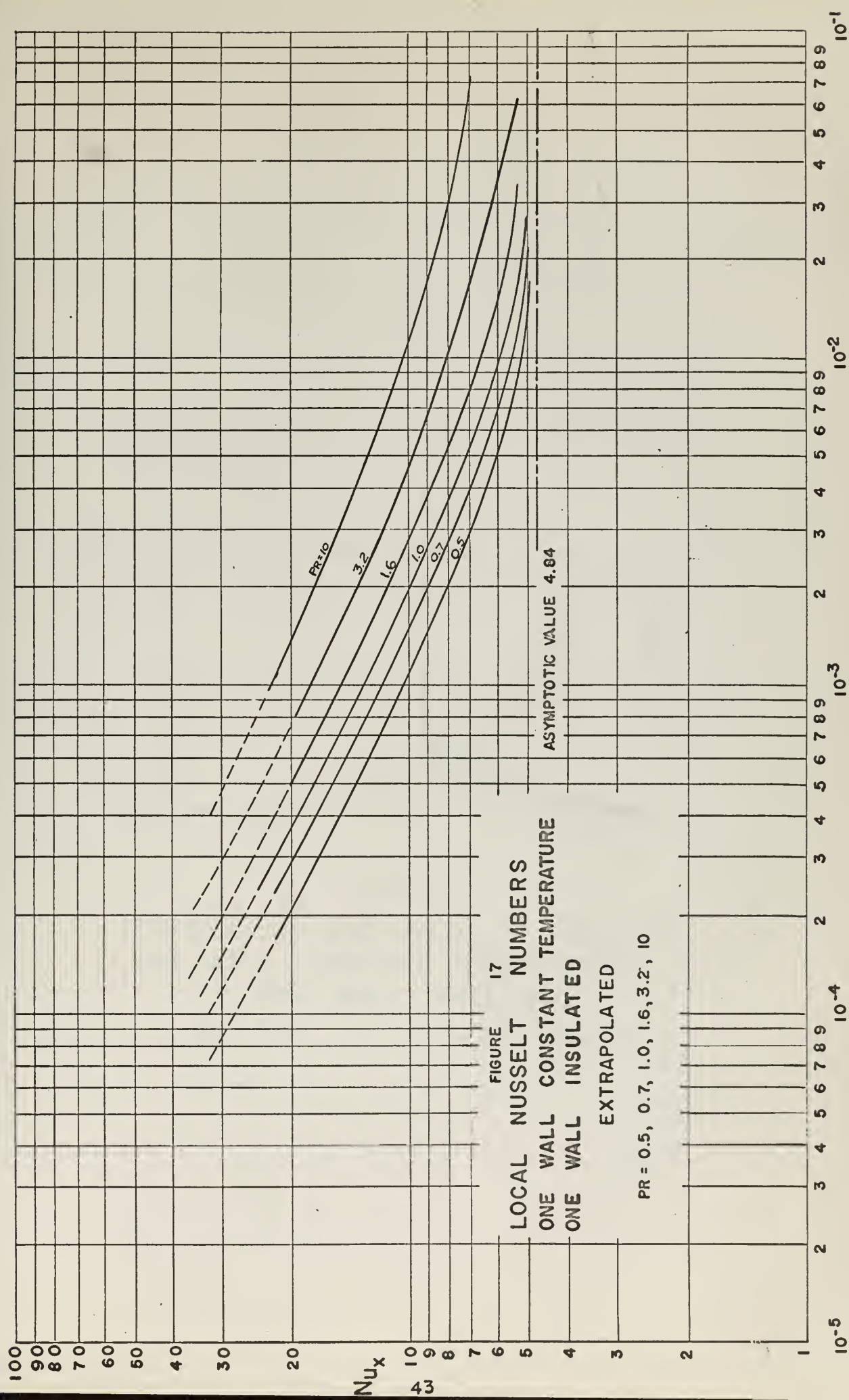


FIGURE 17
 LOCAL NUSSOLT NUMBERS
 ONE WALL CONSTANT TEMPERATURE
 ONE WALL INSULATED
 EXTRAPOLATED

$PR = 0.5, 1.0, 1.6, 3.2, 10$

ASYMPTOTIC VALUE 4.84

L



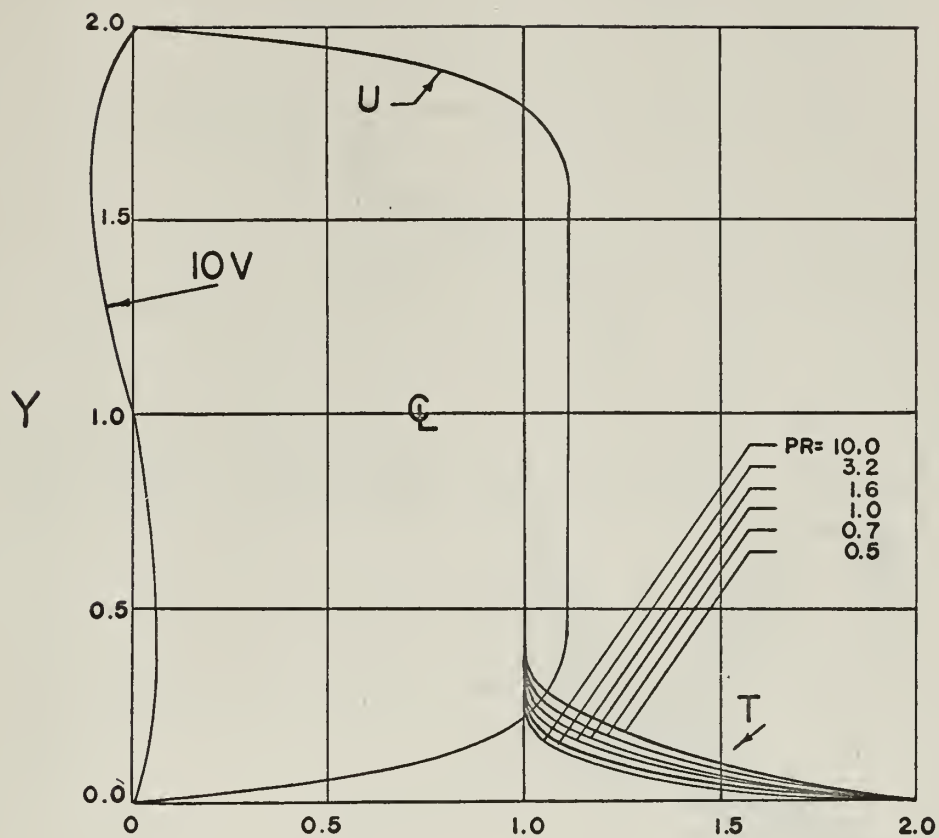
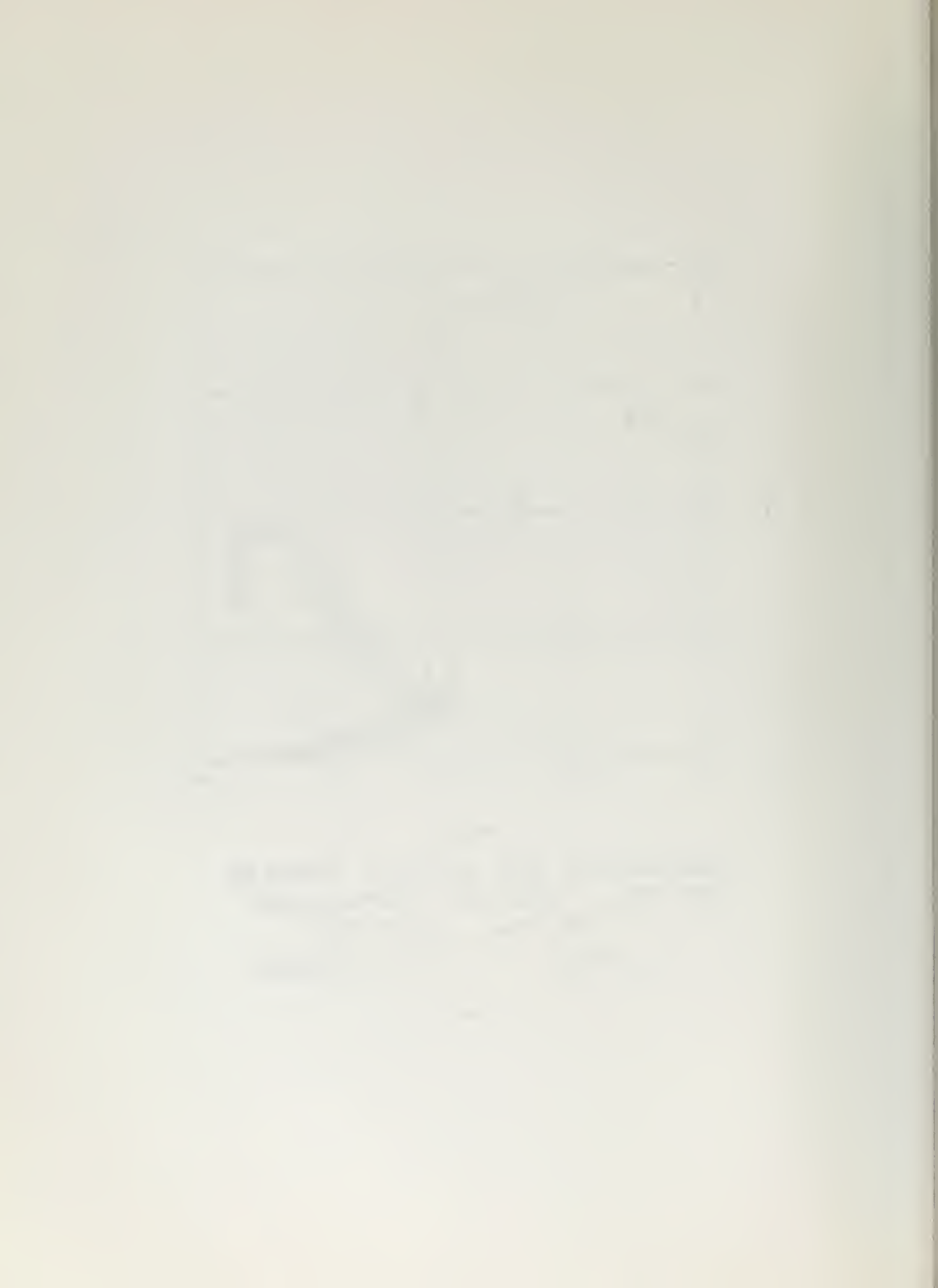


FIGURE 18

TEMPERATURE AND VELOCITY PROFILES
 ONE WALL CONSTANT TEMPERATURE
 ONE WALL INSULATED

$X = 0.005$

$L = 0.0003125$



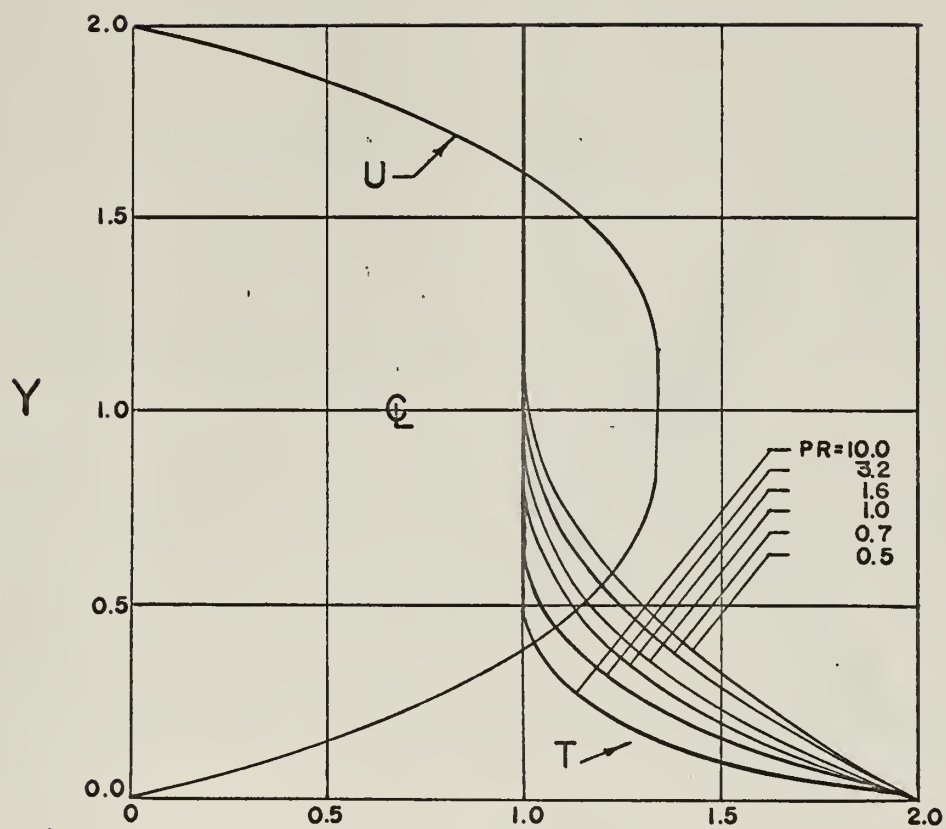


FIGURE 19
 TEMPERATURE AND VELOCITY PROFILES
 ONE WALL CONSTANT TEMPERATURE
 ONE WALL INSULATED
 $X=0.050$ $L=0.003125$



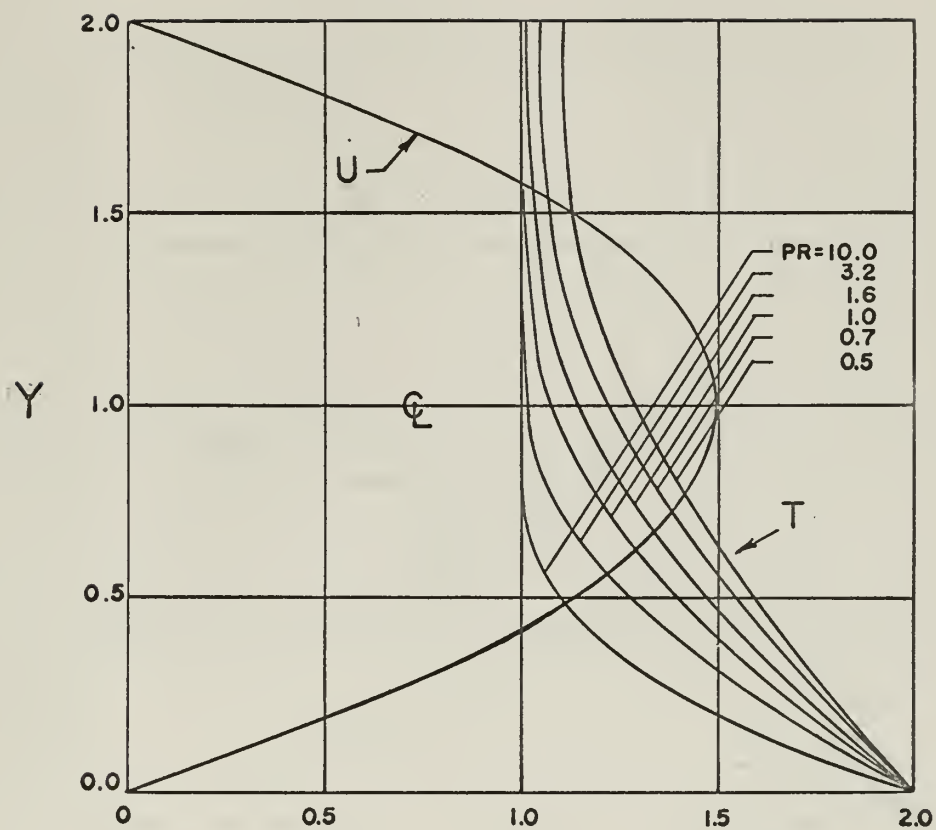


FIGURE 20

TEMPERATURE AND VELOCITY PROFILES
 ONE WALL CONSTANT TEMPERATURE
 ONE WALL INSULATED
 $X=0.250$ $L=0.0156$



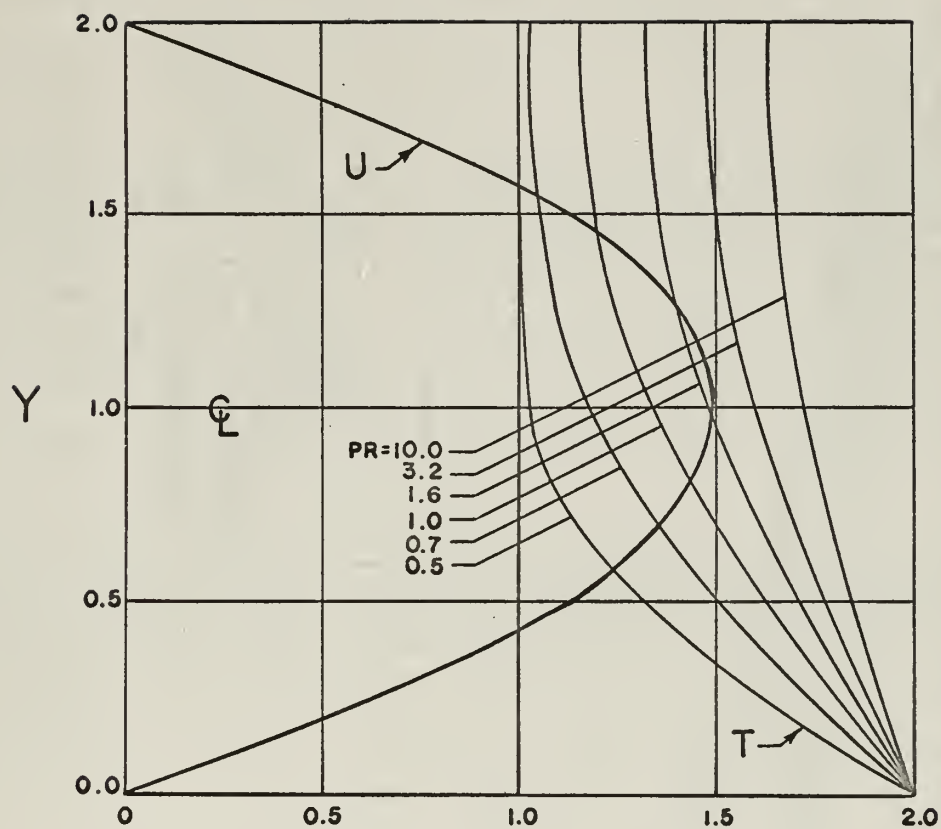
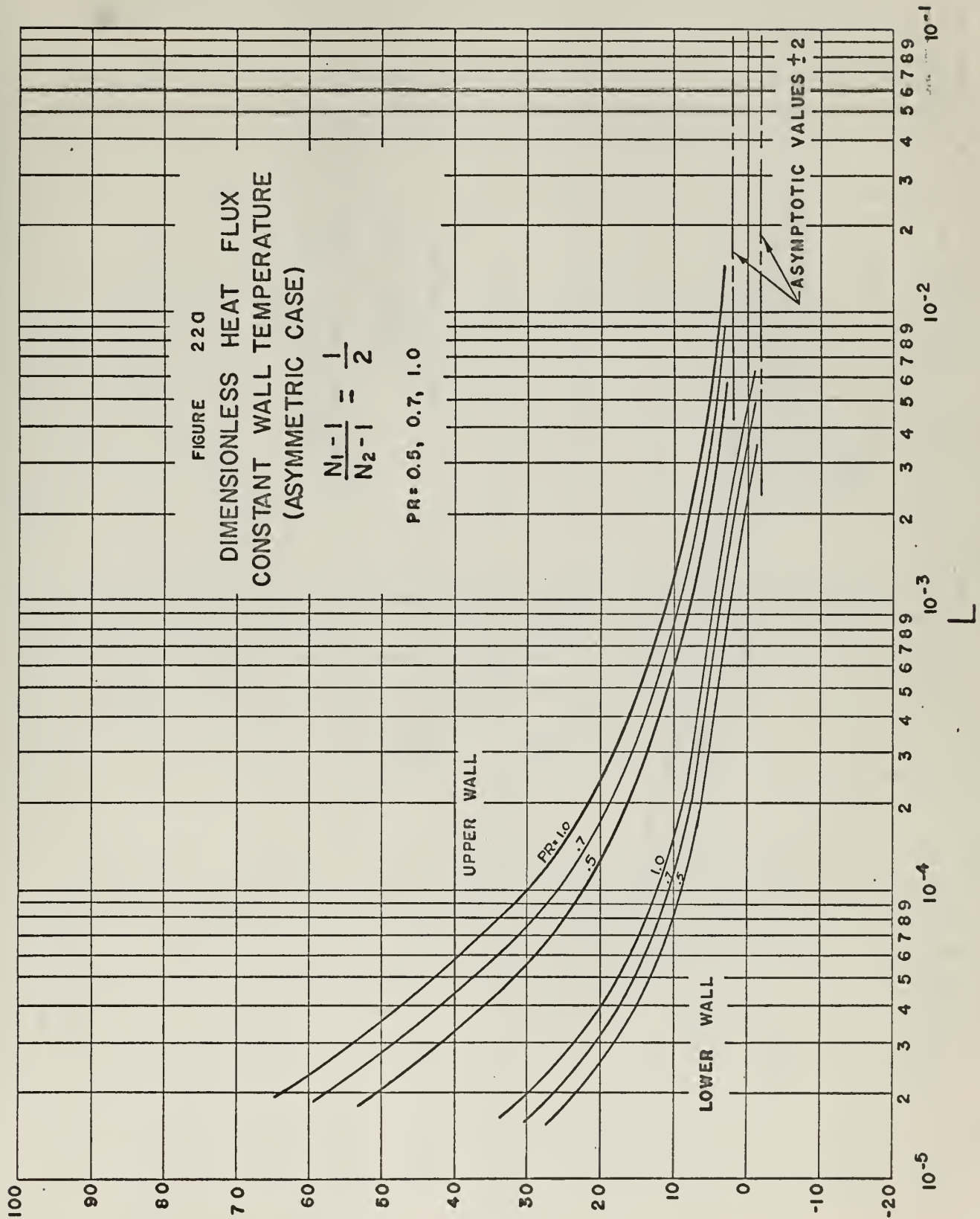


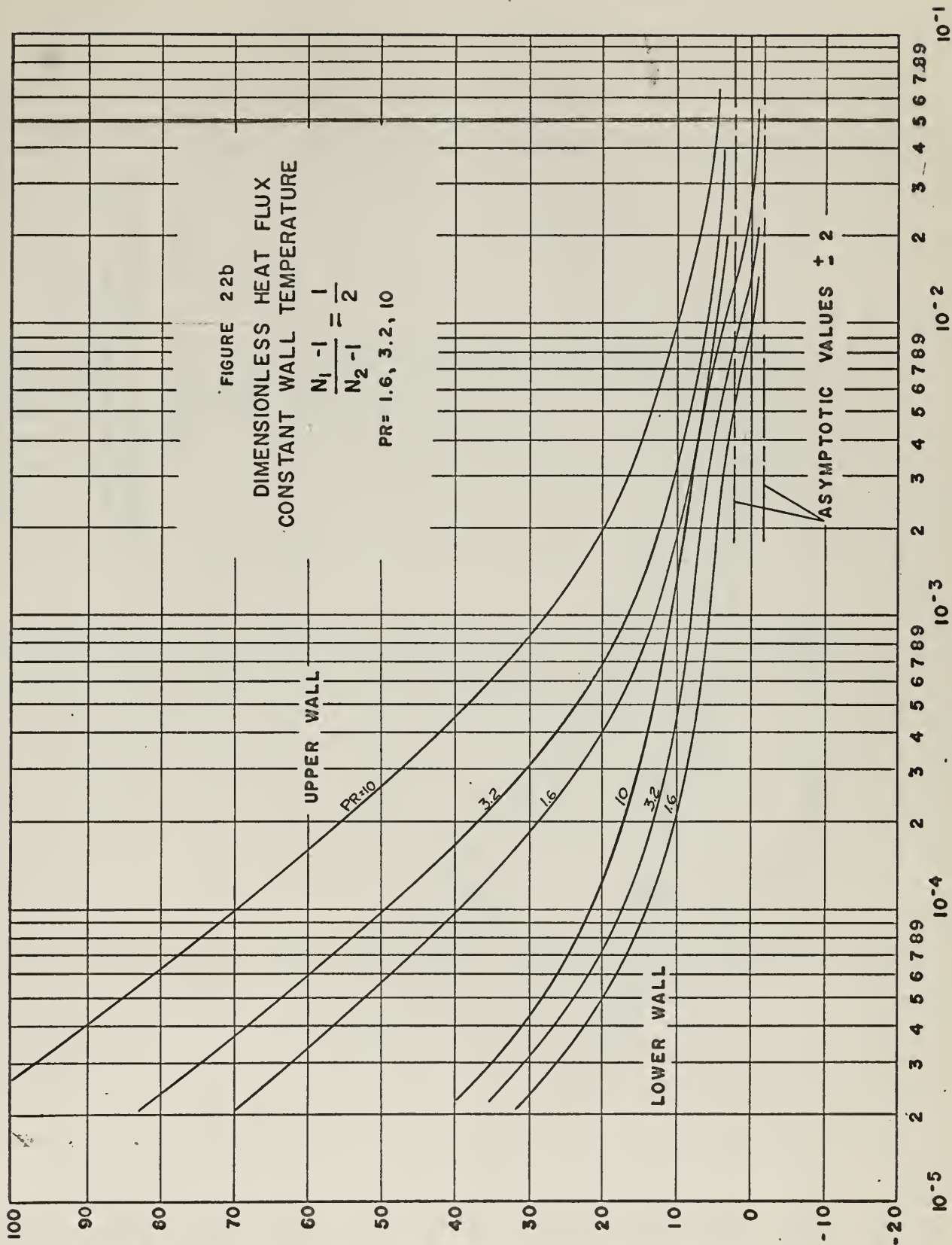
FIGURE 21
 TEMPERATURE AND VELOCITY PROFILES
 ONE WALL CONSTANT TEMPERATURE
 ONE WALL INSULATED
 $X = 1.00$ $L = 0.0625$



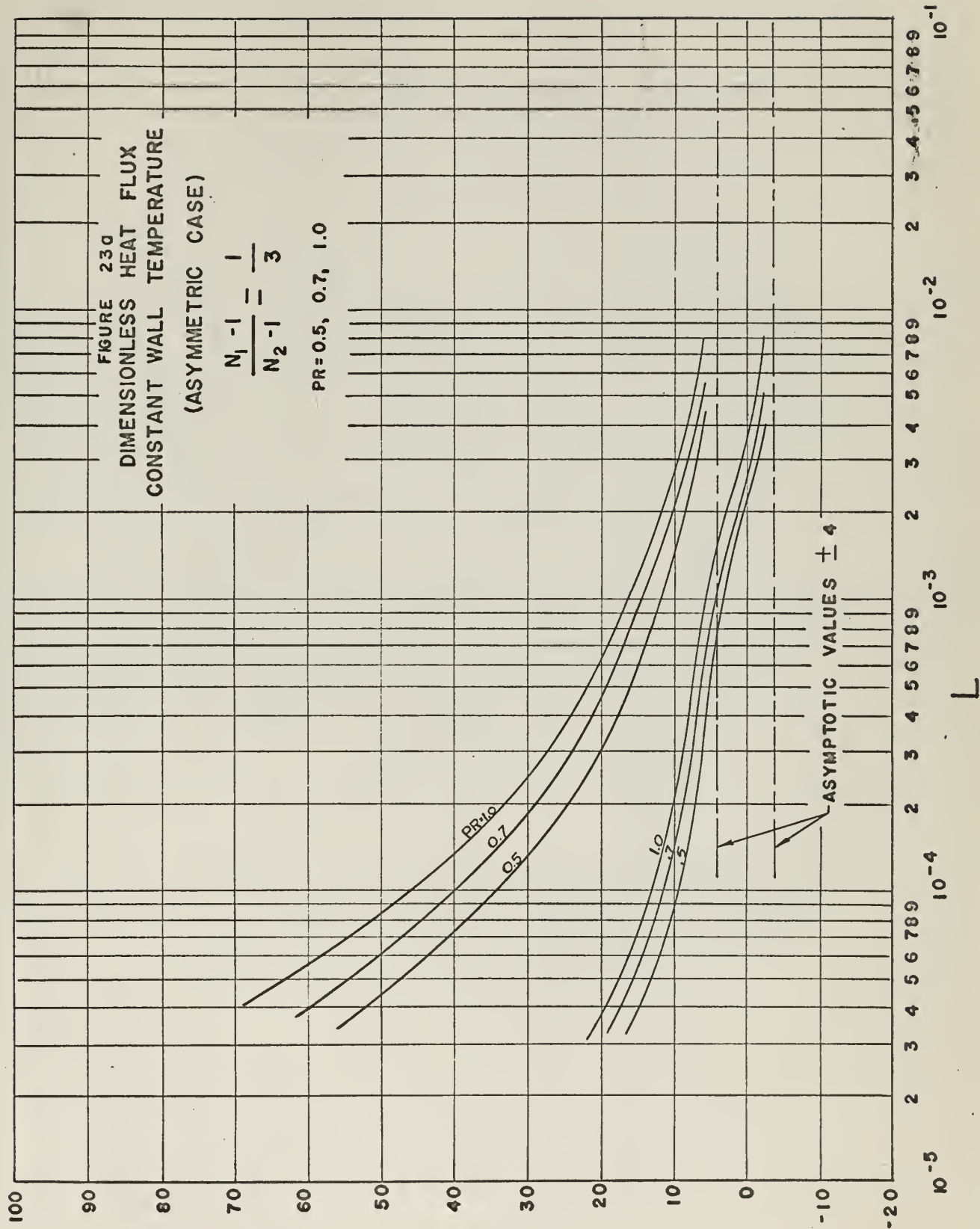
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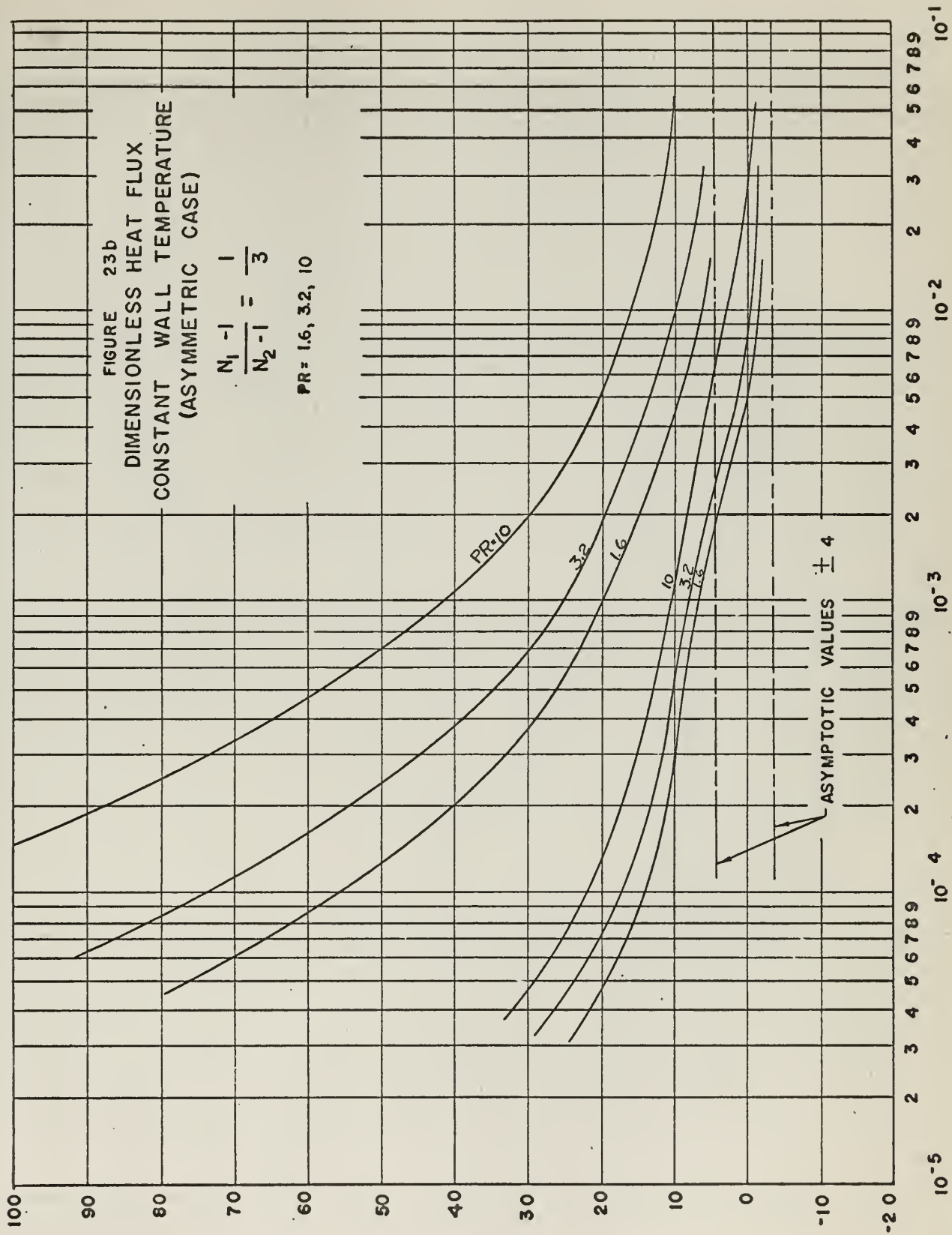


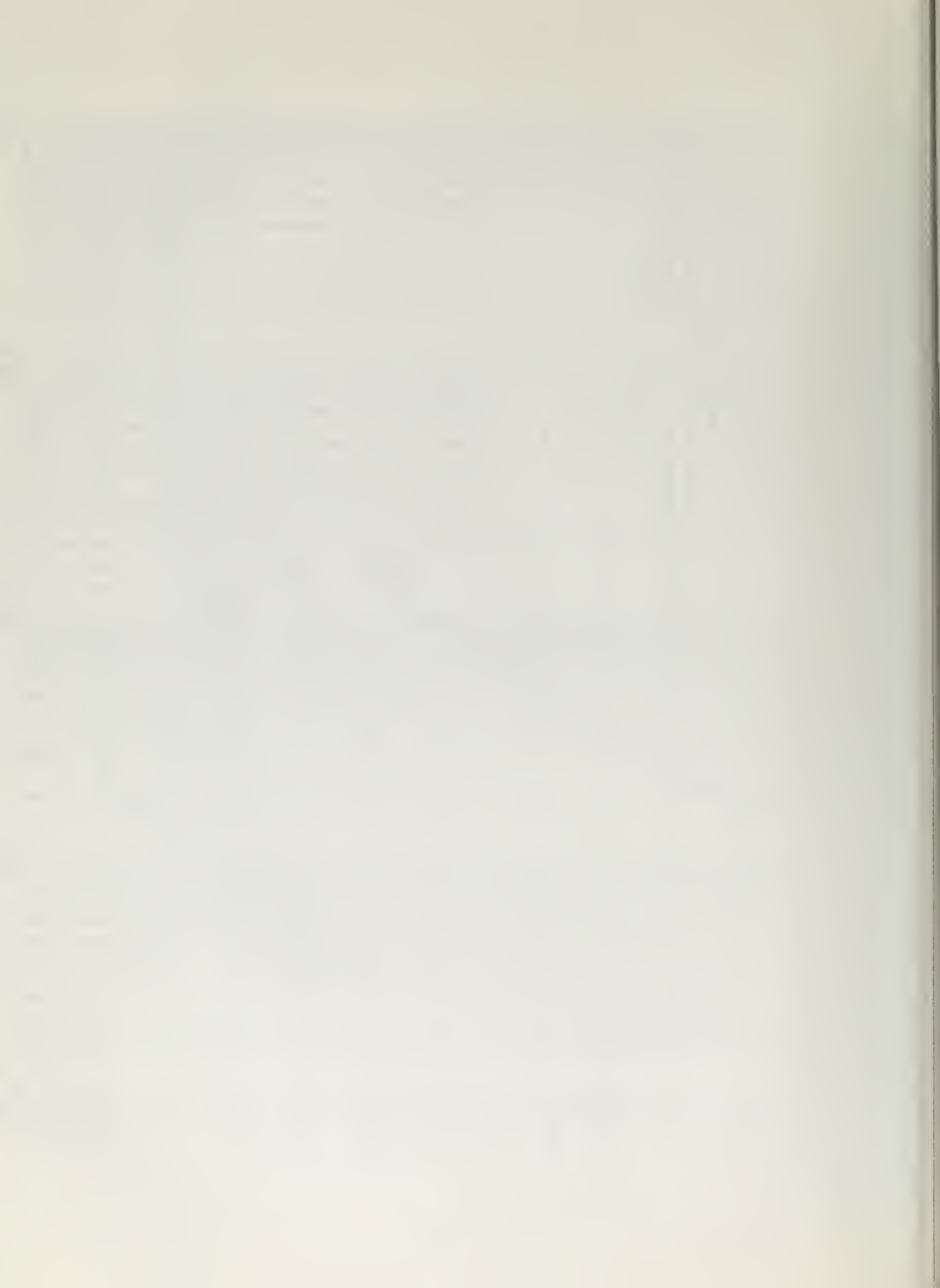


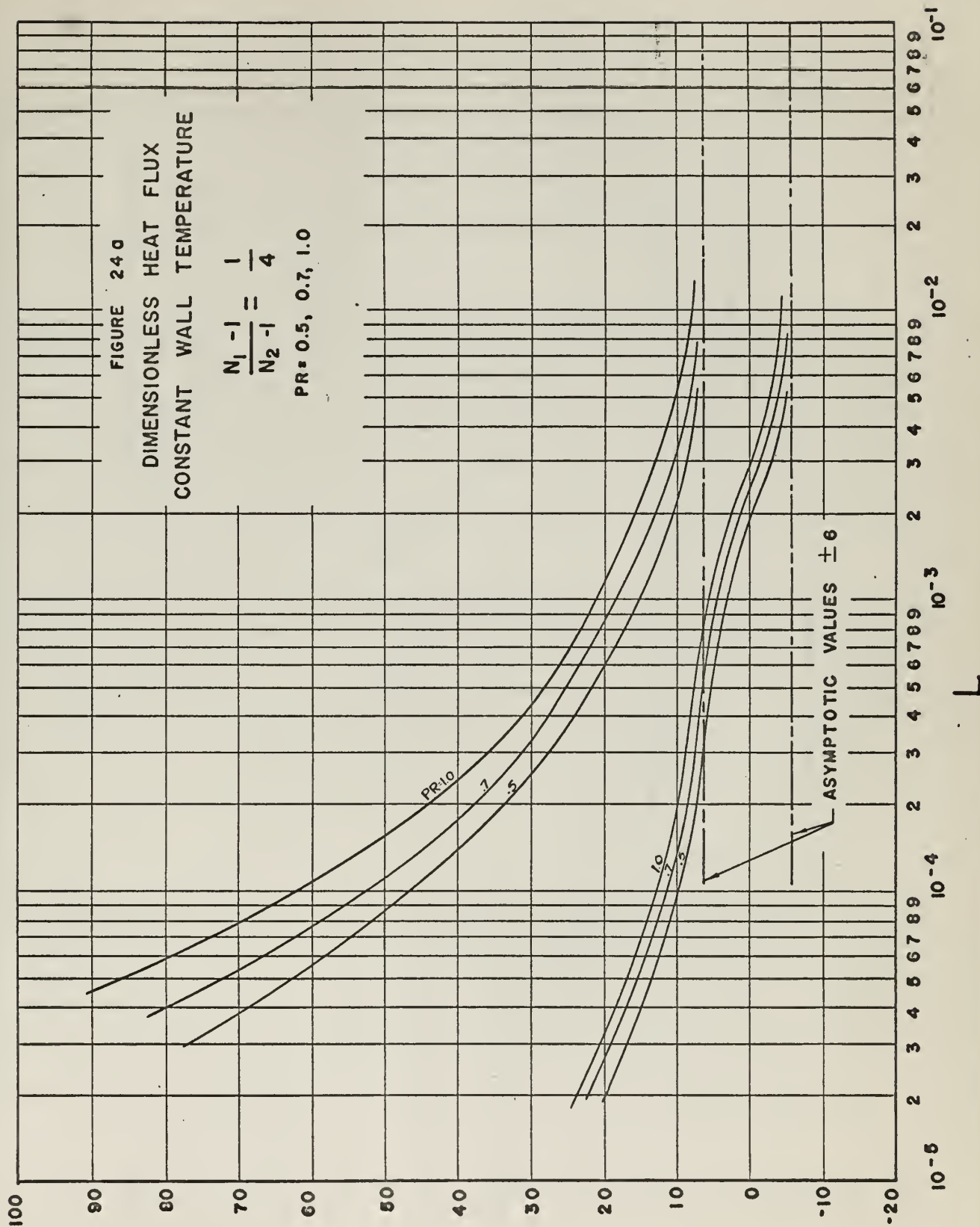


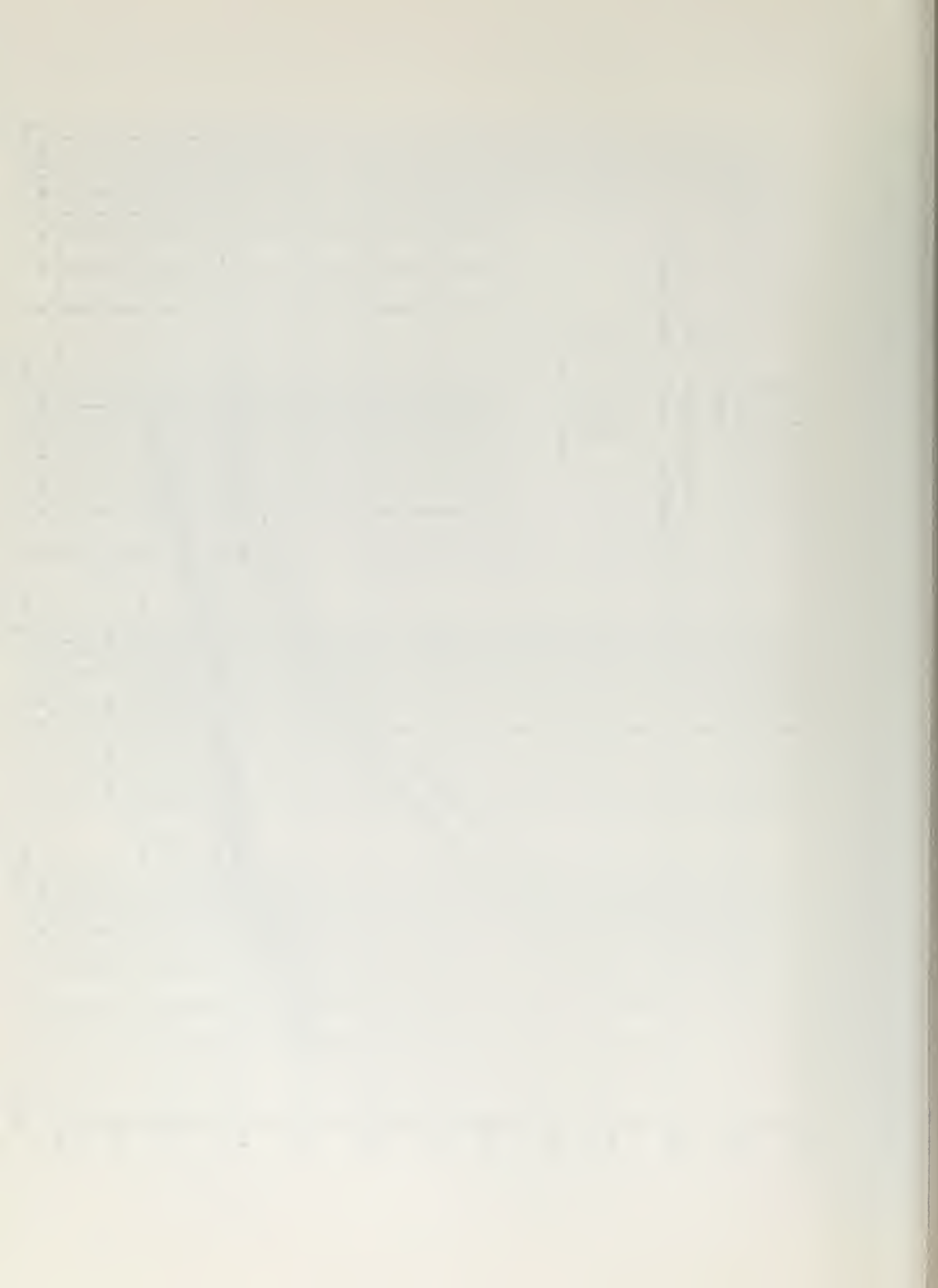


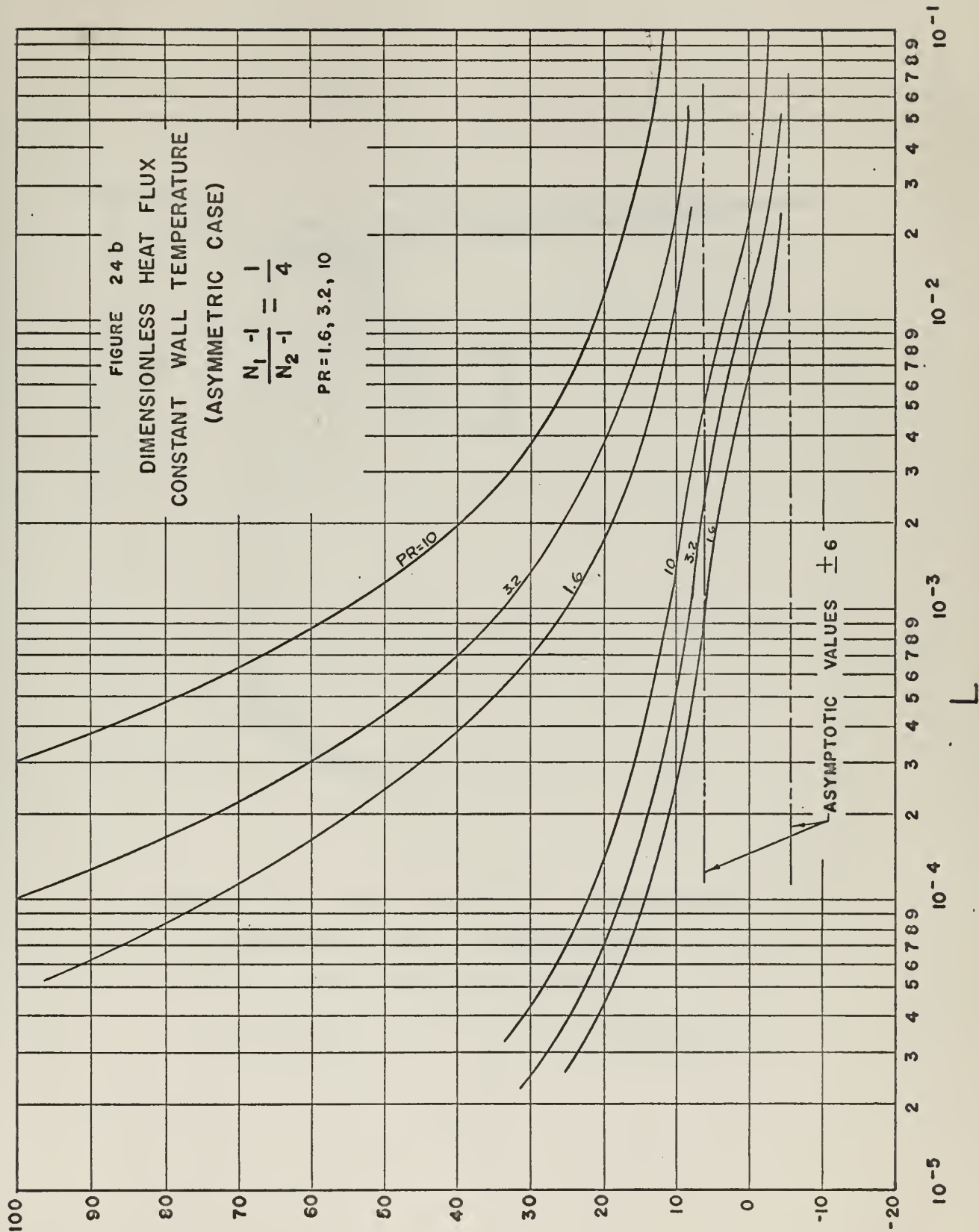


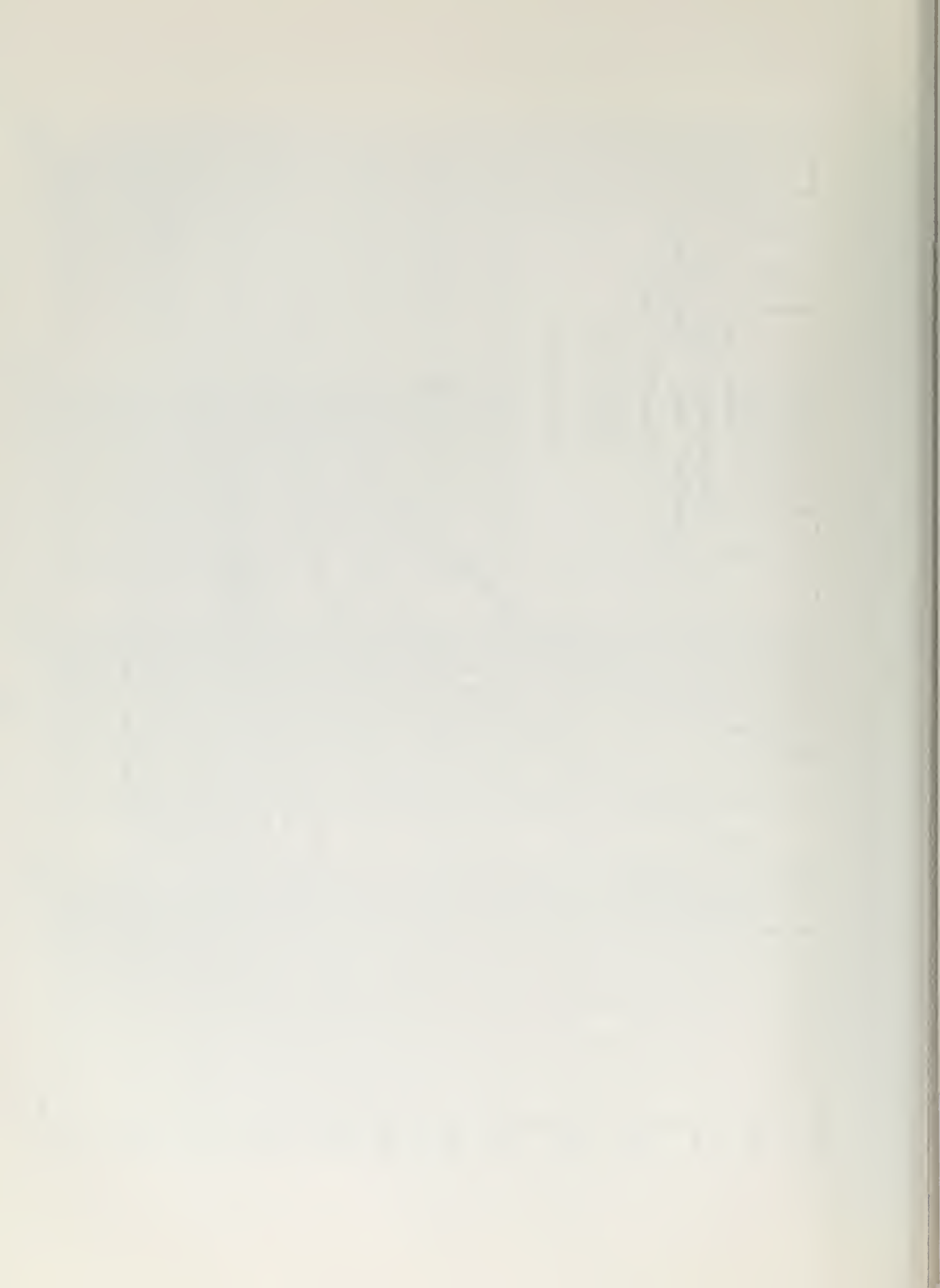












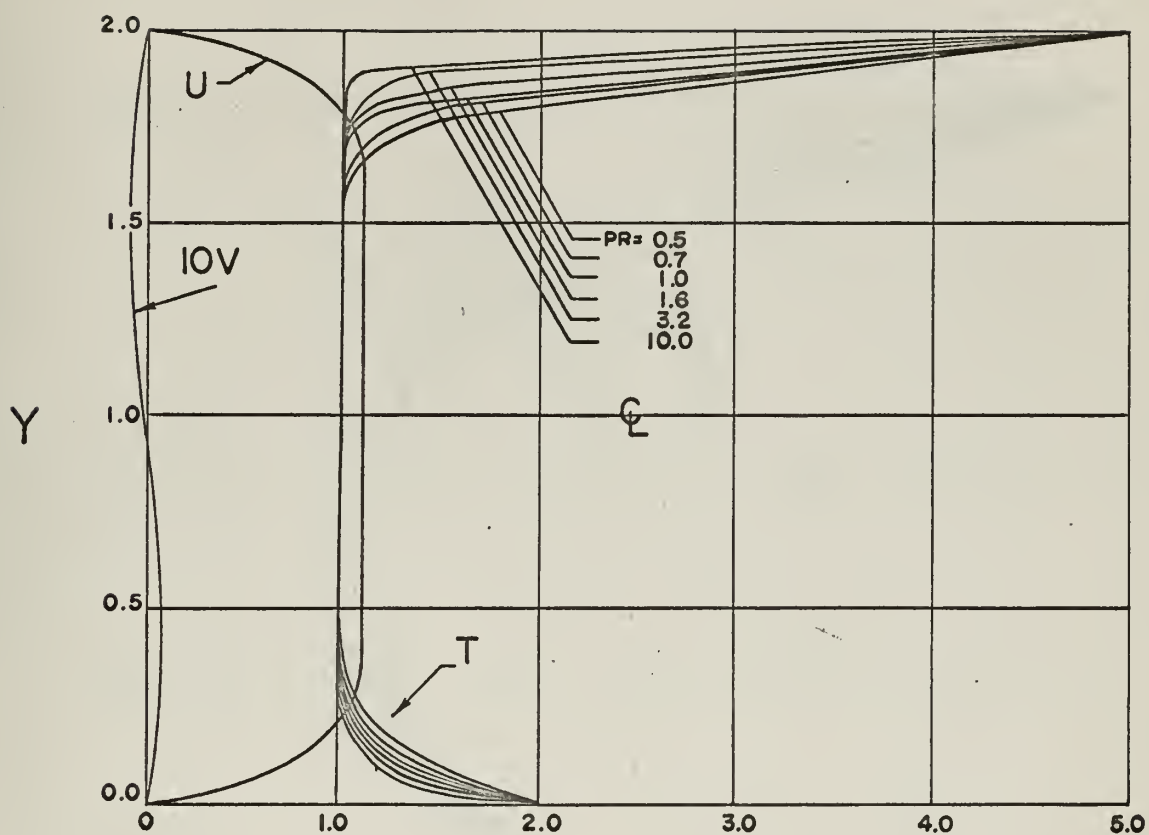
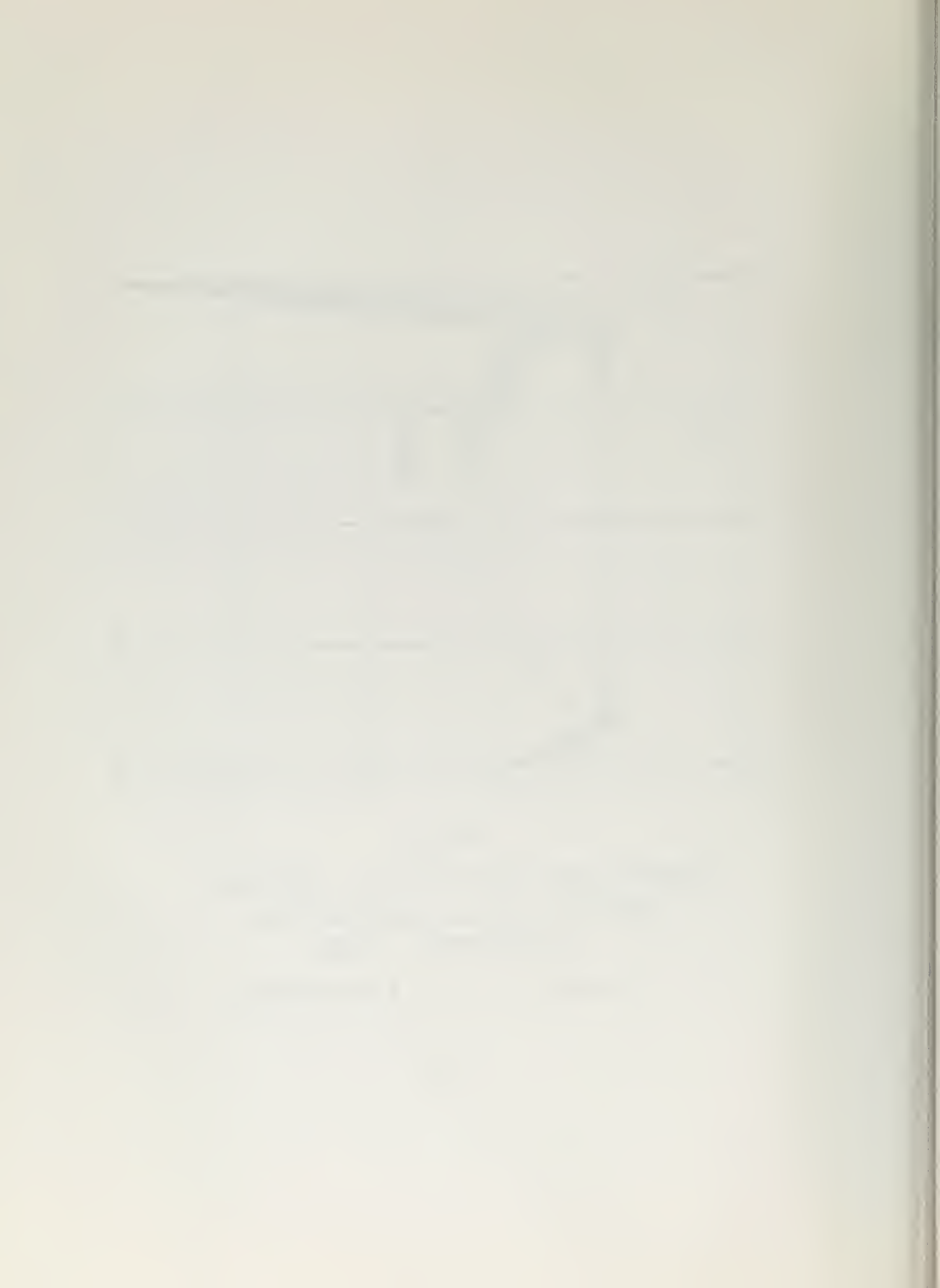


FIGURE 25

TEMPERATURE AND VELOCITY PROFILES
CONSTANT WALL TEMPERATURE
(ASYMMETRIC CASE)

$X=0.005$

$L=0.0003125$



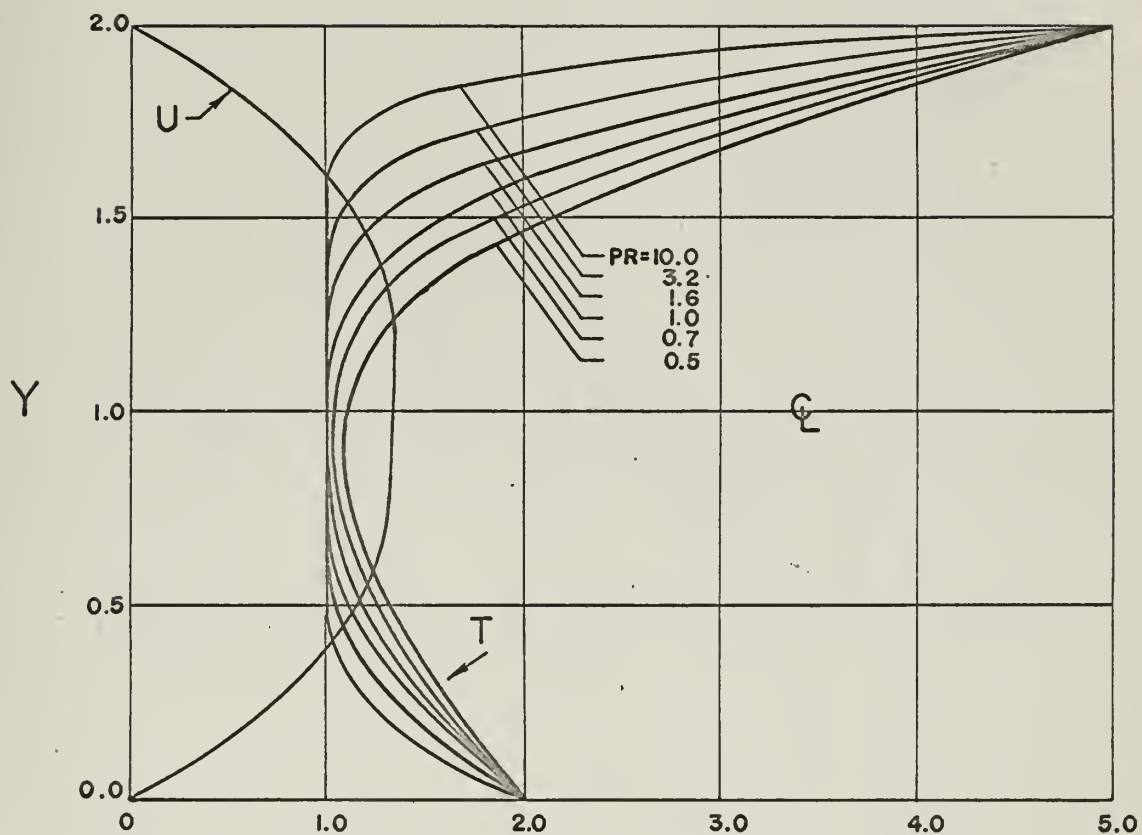
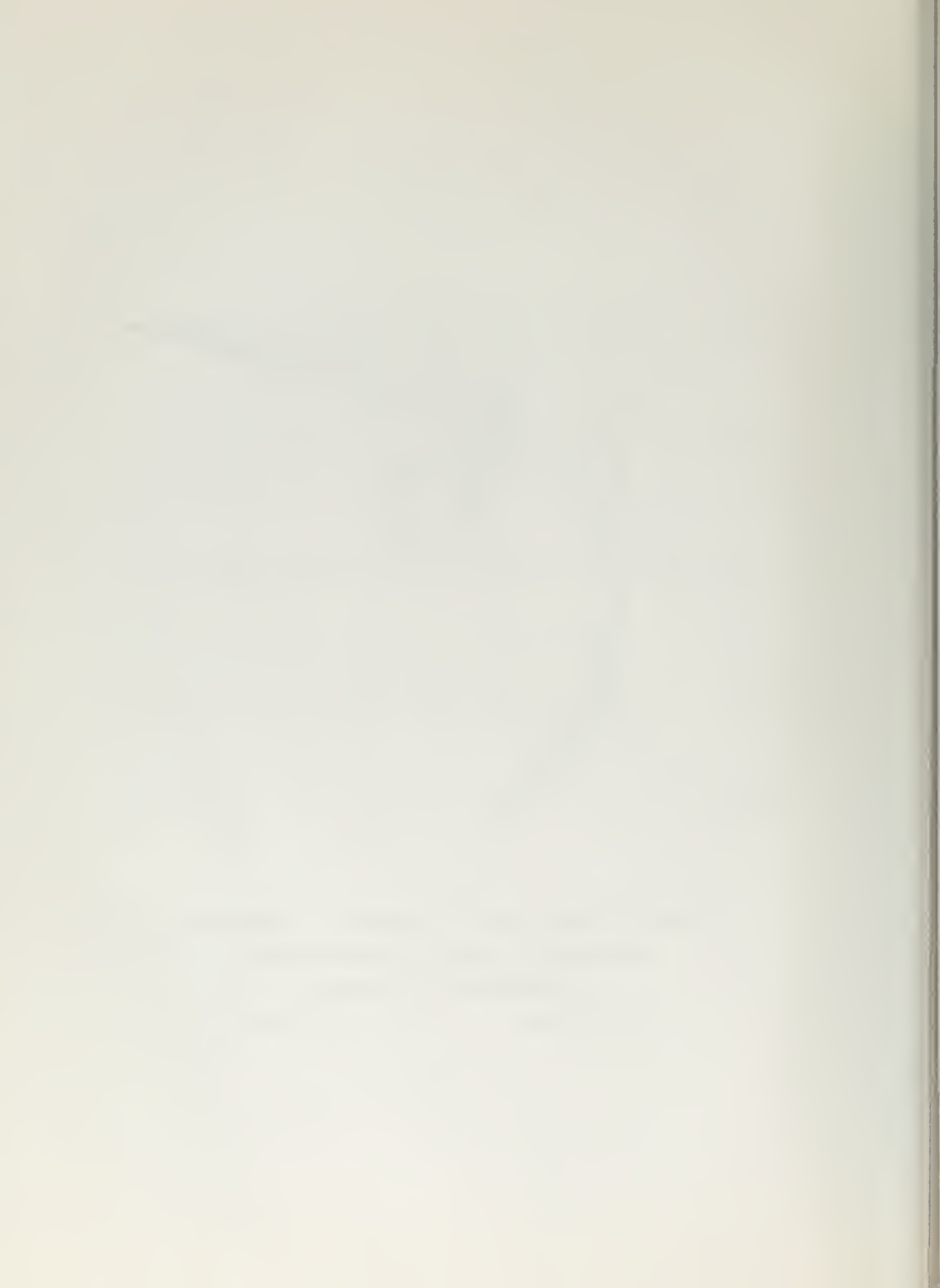


FIGURE 26

TEMPERATURE AND VELOCITY PROFILES
CONSTANT WALL TEMPERATURE
(ASYMMETRIC CASE)

$X = 0.050$

$L = 0.003125$



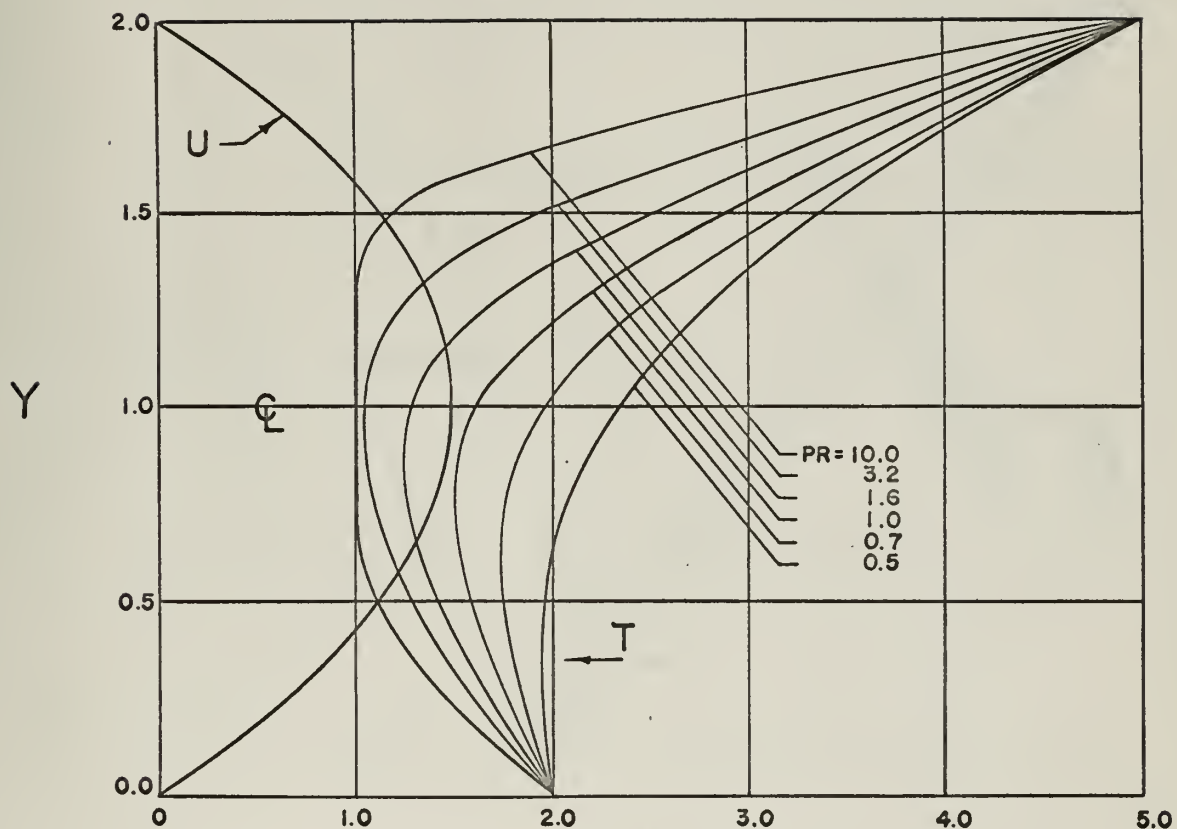
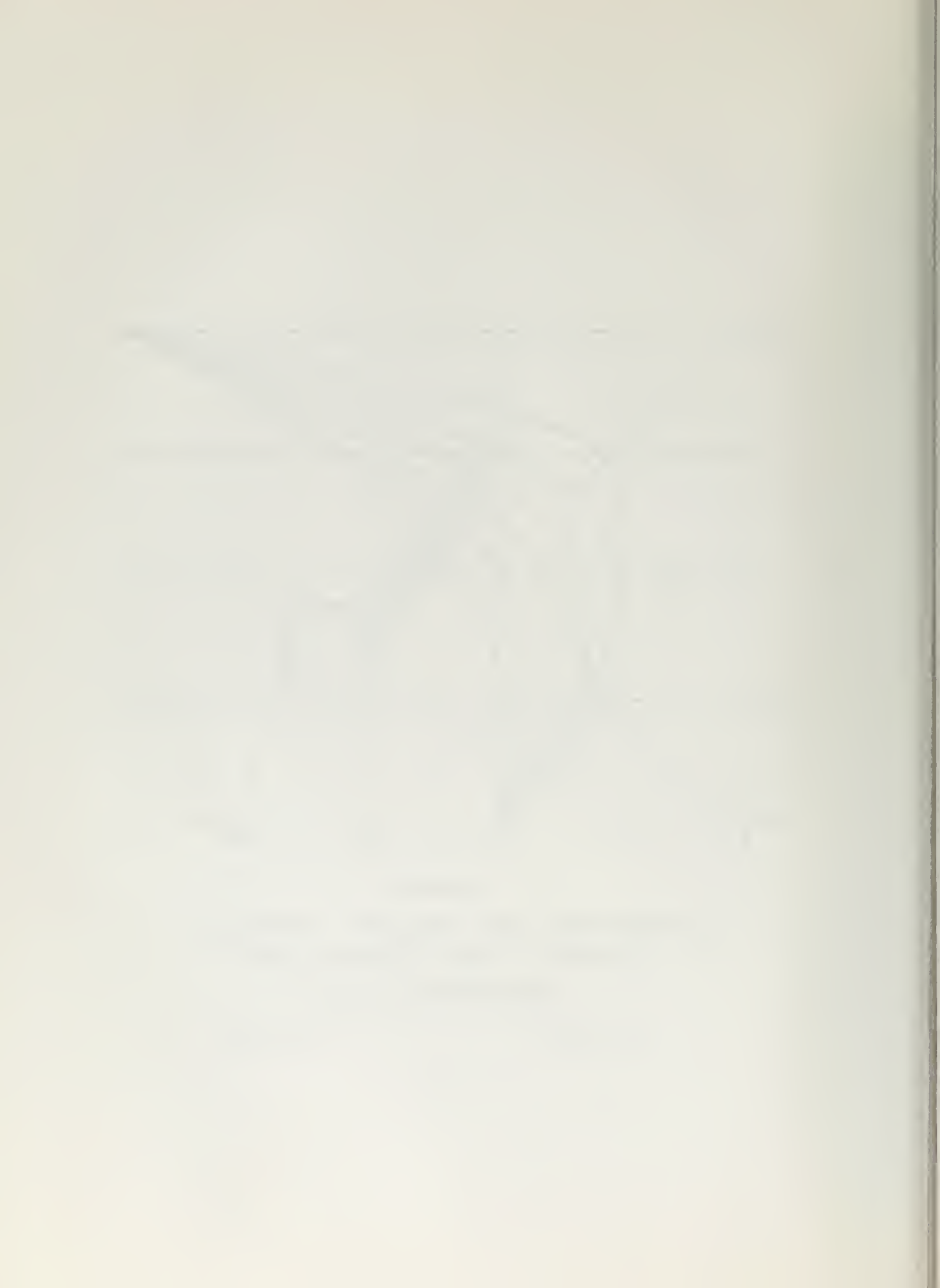


FIGURE 27

TEMPERATURE AND VELOCITY PROFILES
CONSTANT WALL TEMPERATURE
(ASYMMETRIC CASE)

$X = 0.250$

$L = 0.0156$



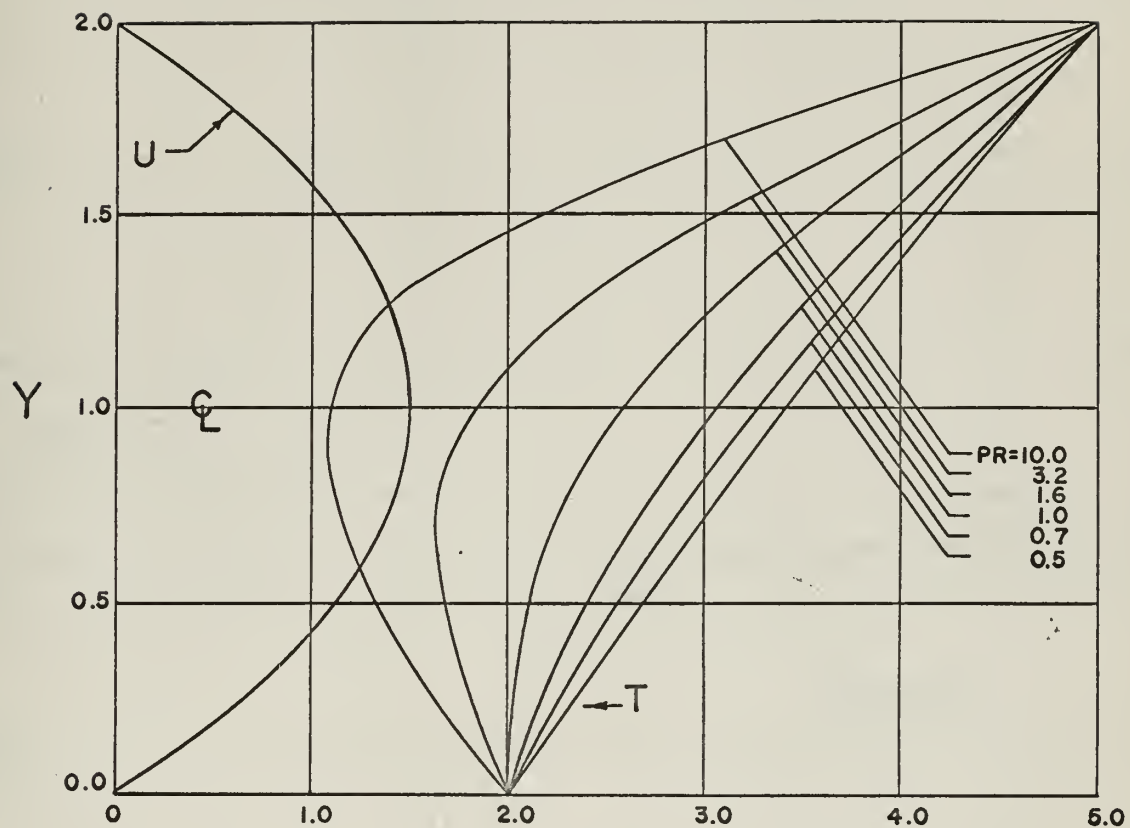
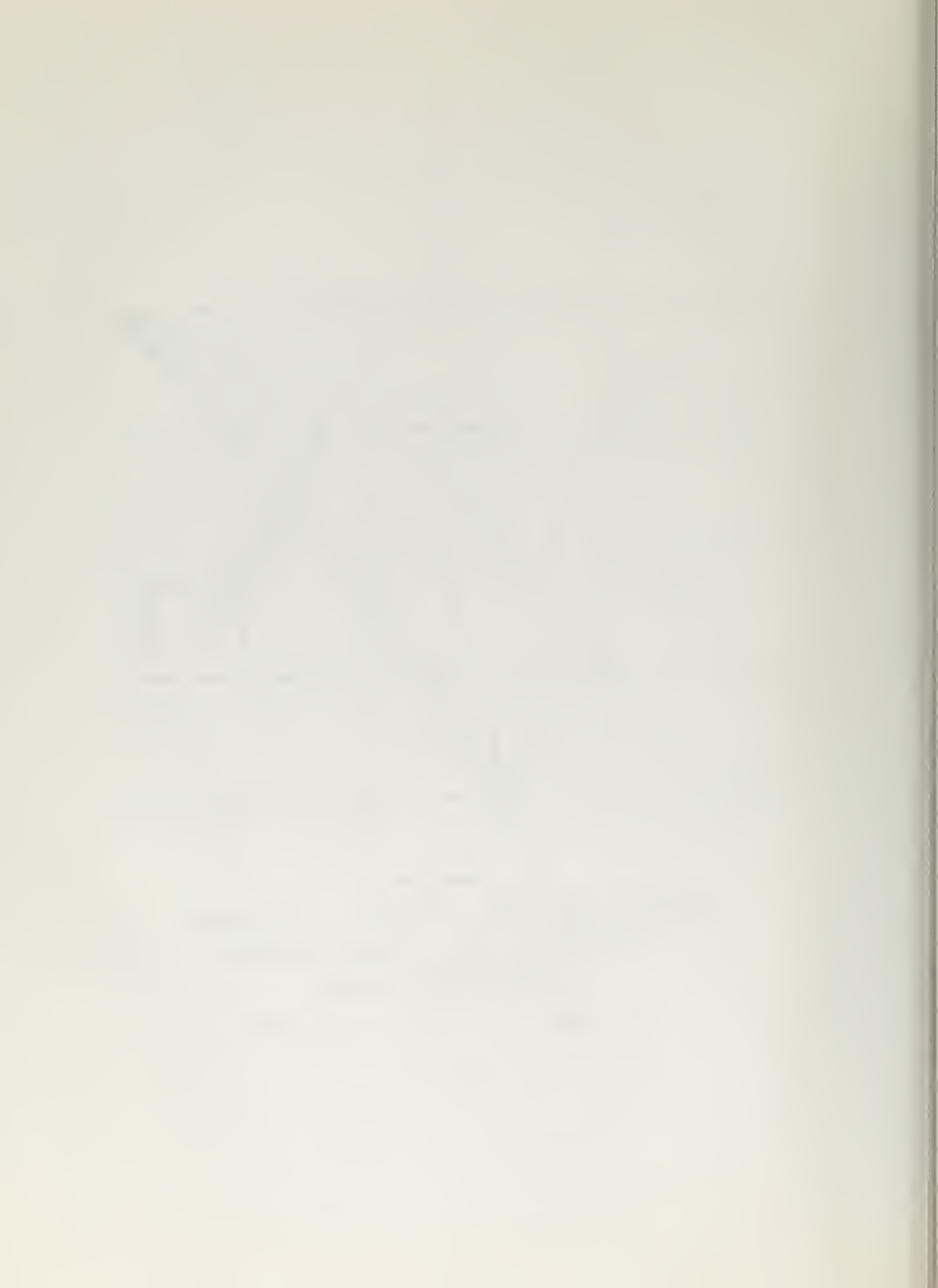


FIGURE 28

TEMPERATURE AND VELOCITY PROFILES
 CONSTANT WALL TEMPERATURE
 (ASYMMETRIC CASE)

$X = 1.00$

$L = 0.0625$



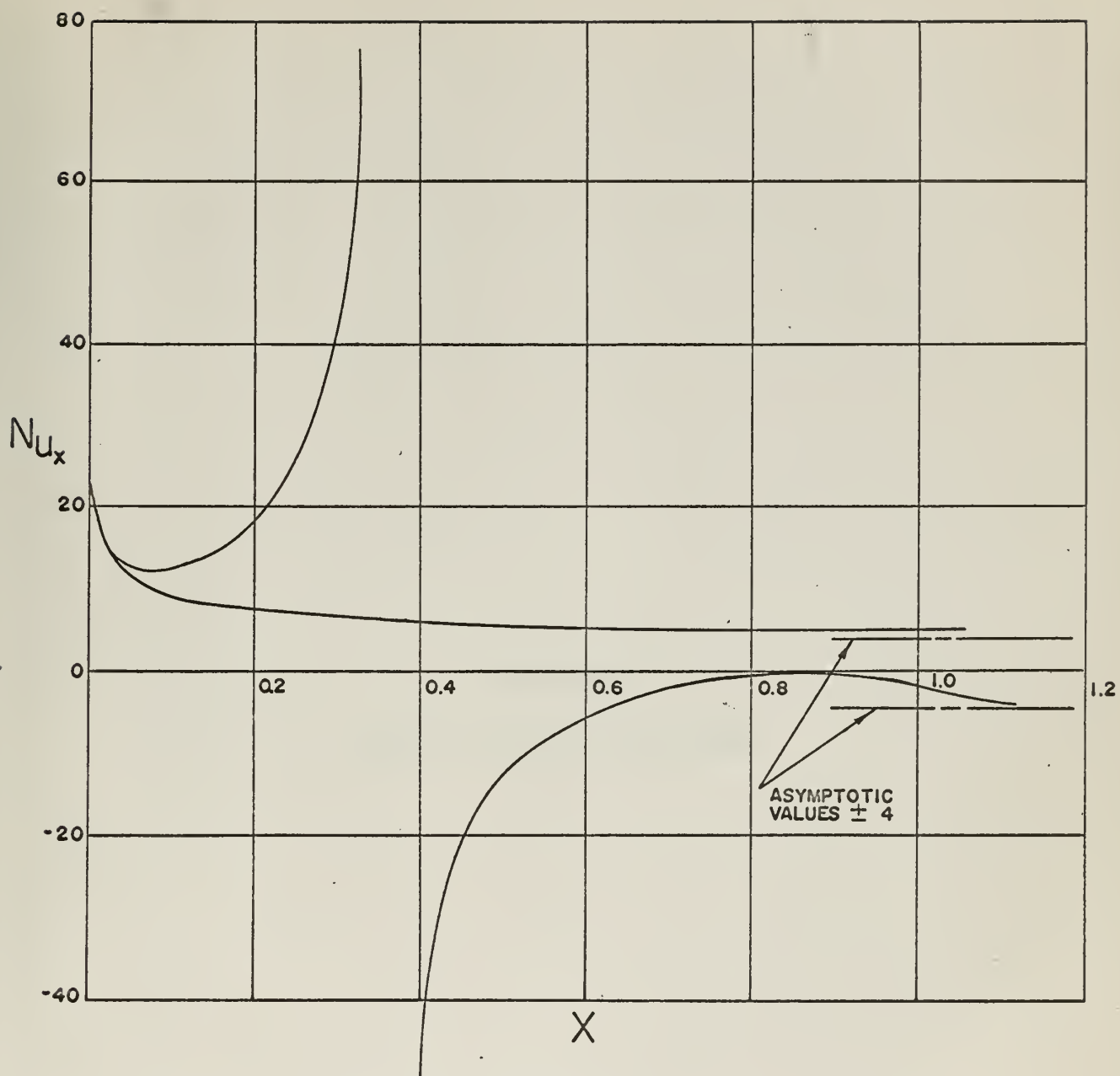
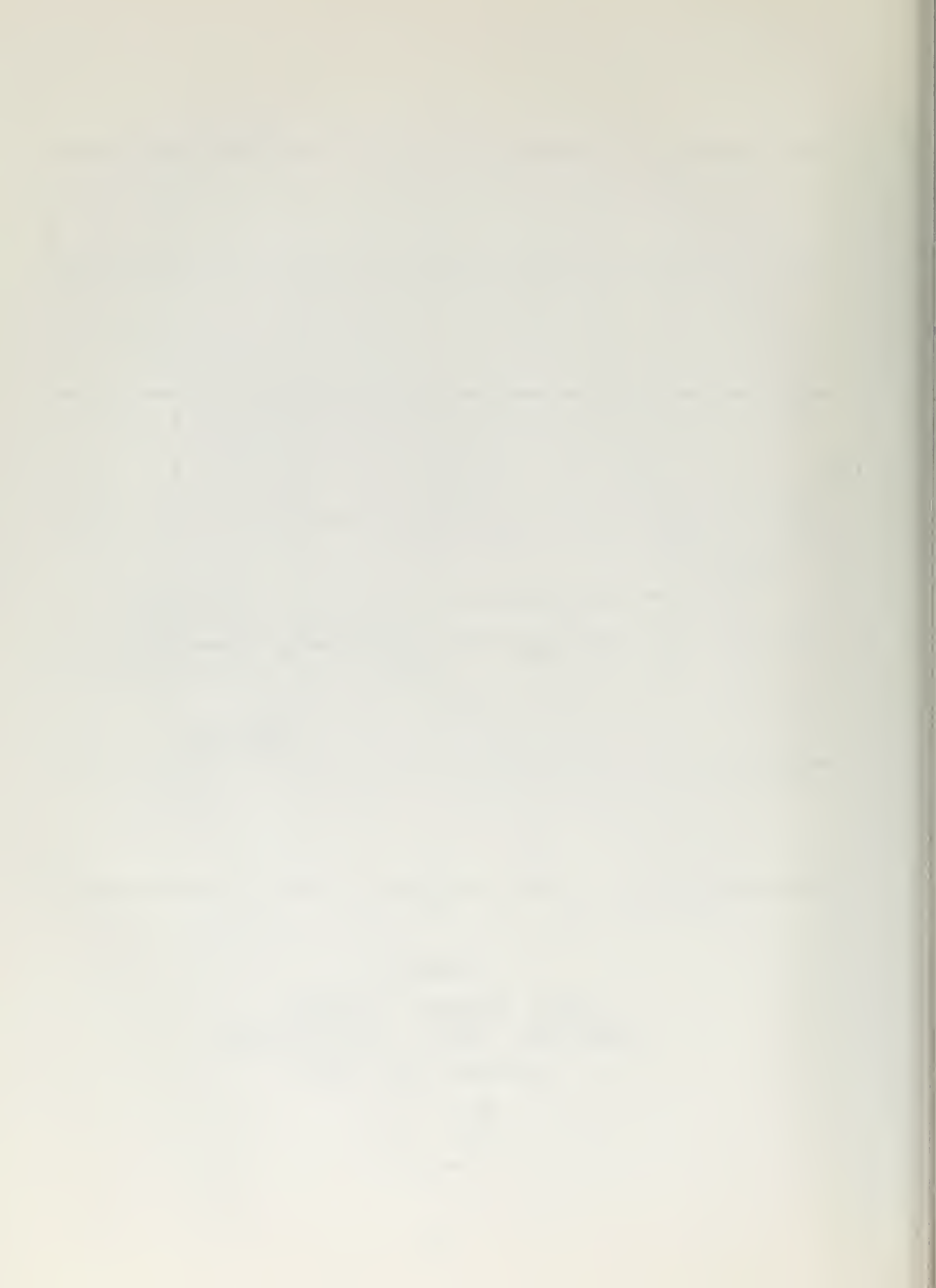


FIGURE 29

LOCAL NUSSLETT NUMBER
CONSTANT WALL TEMPERATURE
(ASYMMETRIC CASE)

$$\frac{N_1 - 1}{N_2 - 1} = \frac{1}{4}$$

PR = 0.7



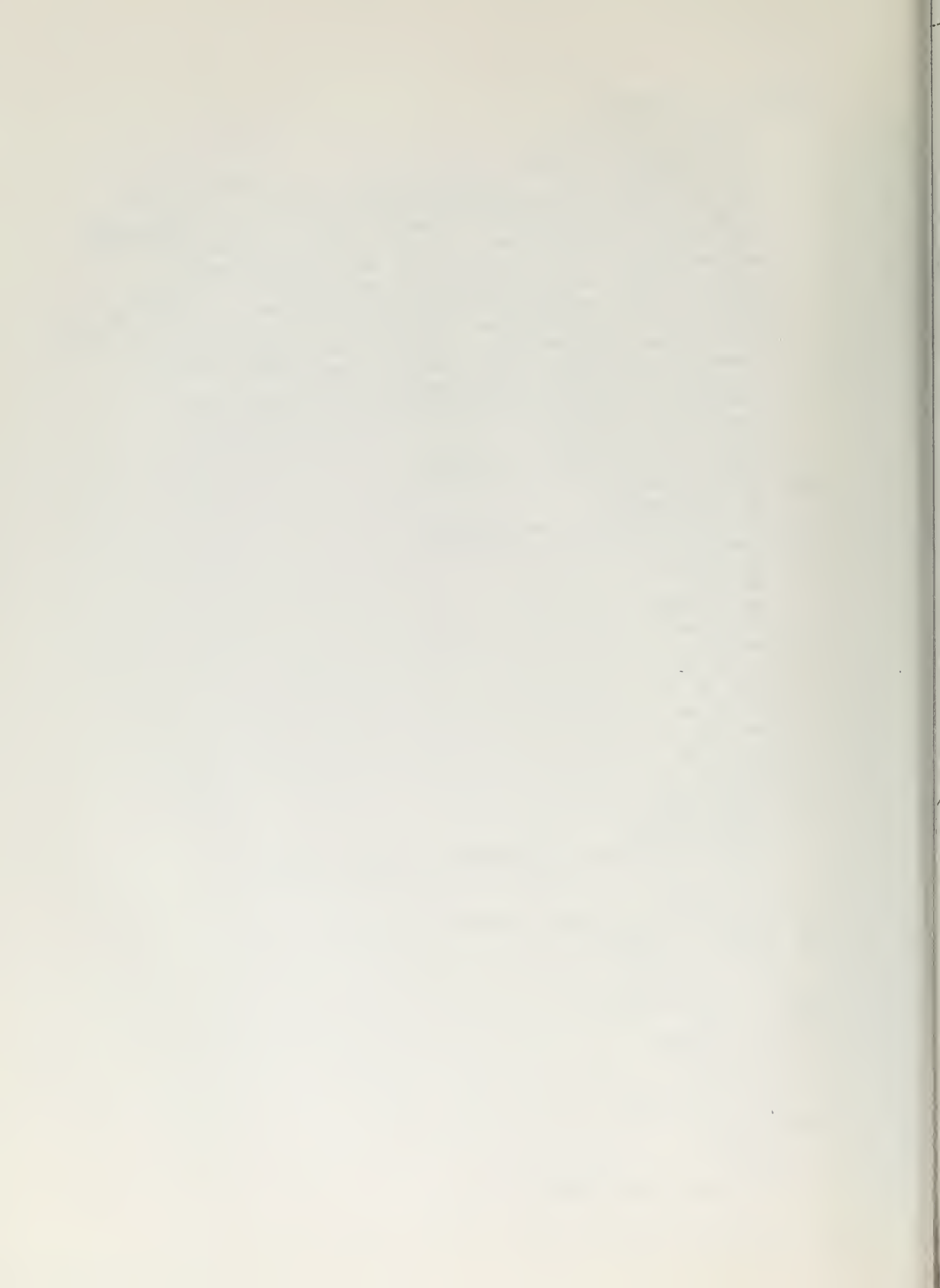
APPENDIX A
SAMPLE FORTRAN 60 PROGRAM



```

..JOB0191F,LUNDBERG
PROGRAM HEATX23
C   CONSTANT HEAT INPUT
C   BODOIA VELOCITY DATA
C   THIS PROGRAM USES A REDUCED GRID OF X=.0001,Y=.05
C   FOR THE FIRST 20 STREAMWISE CALCULATIONS.
C   GRAPH OUTPUTS OF NUSSELT NUMBER VERSUS LENGTH PARAMETER
C   FOR PRANDTL NUMBERS OF 0.5,0.7,1.0,1.6,3.2,AND 10,
C   ARE OBTAINED TO 50 PER CENT OF THE HYDRODYNAMIC
C   DEVELOPMENT. PRINT OUTPUT INCLUDES NUSSELT NUMBER,
C   SOPE OF THE TEMPERATURE PROFILE AT THE WALL , AND THE
C   SLOPE OF THE TEMPERATURE PROFILE AT THE WALL,AND THE MEAN
C   MIXED MEAN TEMPERATURE. IN ADDITION, VELOCITY AND
C   TEMPERATURE VALUES ARE PRINTED AT EACH GRID POINT AT
C   SELECTED STREAMWISE LOCATIONS.
DIMENSION T(30,23),U(30,23),V(30,23),Y(23),X(500)
1,XN(500),ITITLE(12)
READ 500,(ITITLE(I), I=1,6)
READ 500,(ITITLE(I), I=7,12)
500 FORMAT (6A8)
DO 750 KK=1,6
GO TO(40,41,42,43,44,45),KK
40 PR=0.7
GO TO 46
41 PR=0.5
GO TO 888
GO TO 46
42 PR=1.0
GO TO 46
43 PR=1.6
GO TO 46
44 PR=3.2
GO TO 46
45 PR=10.
46 PRINT 523
523 FORMAT (1H1)
PRINT 17, PR
17 FORMAT (//////10X,17H PRANDTL NUMBER =,F5.3)
L=1
GO TO 150
700 READ 701, ((U(I,J), J=13,22), I=2,22)
701 FORMAT (10F6.4)
DO 710 I=2,22
DO 711 J=1,12
711 U(I,J)=U(I,13)
710 CONTINUE
DELX=.0001
DELY=0.05
R=1000.
DO 110 L=2,21
150 UTM=0.0
UM=0.0
XL=L
X(L)=(XL-1.0)*0.0001

```




```

DO 111 J=3,22
XJ=J
Y(L)=(XJ-3.0)*0.05
Y(23)=1.0
U(1,J)=1.
U(1,23)=1.0
V(1,J-1)=0.0
V(1,J)=0.
T(1,J)=1.0
T(1,J+1)=1.0
T(1,J-1)=1.0
T(L,1)=T(L,5)
U(L,1)=U(L,5)
U(L,23)=0.0
U(L,2)=U(L,4)
V(L,3)=0.0
V(L,2)=V(L,4)
C THERMAL BOUNDARY CONDITIONS
T(L,23)=T(L,22)+.05
T(L,2)=T(L,4)
IF(L-1) 109,109,108
108 V(L,J)= V(L,J-1) + DELY/(2.*DELX*R) * (U(L-1,J)-U(L+1,J))
109 A= DELX/ (U(L,J)*PR*DELY**2)
V(L,23)=0.0
B= (DELX*R*V(L,J)) / (2.*DELY*U(L,J))
T(L+1,J)=(A-B)*T(L,J+1)+(1.-2.*A)*T(L,J)+(A+B)*T(L,J-1)
IF(J-3) 113,113,112
113 UTM=UTM+.5*U(L,J)*T(L,J)
UM=UM+.5*U(L,J)
GO TO 114
112 UTM=UTM+U(L,J)*T(L,J)
UM=UM+U(L,J)
114 TM=UTM/UM
111 CONTINUE
T1= ( T(L,23)-T(L,22)) / DELY
T2= T(L,23) - TM
XN(L)=4.0*T1/T2
131 PRINT 118, X(L),XN(L),T2,T1,TM
118 FORMAT (// 4H X =,F6.4,5X,14H NUSSELT NO. =,F8.5,5X,8H TW-TM =,
1F8.5,5X,8H SLOPE =,F8.5,5X,5H TM =,F8.5)
PRINT119, (T(L,J), J=1,12)
PRINT119, (T(L,J), J=13,23)
119 FORMAT (/9H TEMP. =,12F7.4)
PRINT120, (U(L,J), J=1,12)
PRINT120, (U(L,J), J=13,23)
120 FORMAT (/ 9H X VEL. =,12F7.4)
PRINT121, (V(L,J), J=1,12)
PRINT121, (V(L,J), J=13,23)
121 FORMAT (/ 9H Y VEL. =,12F7.4)
IF(L-1) 700,700,110
110 CONTINUE
CALL DRAW (21,X,XN,MOD,0,LAB,ITITLE,.04,10.,0,0,0,0,6, 6,1, LAST)
DO 720 N=2,11
M=2*N-1

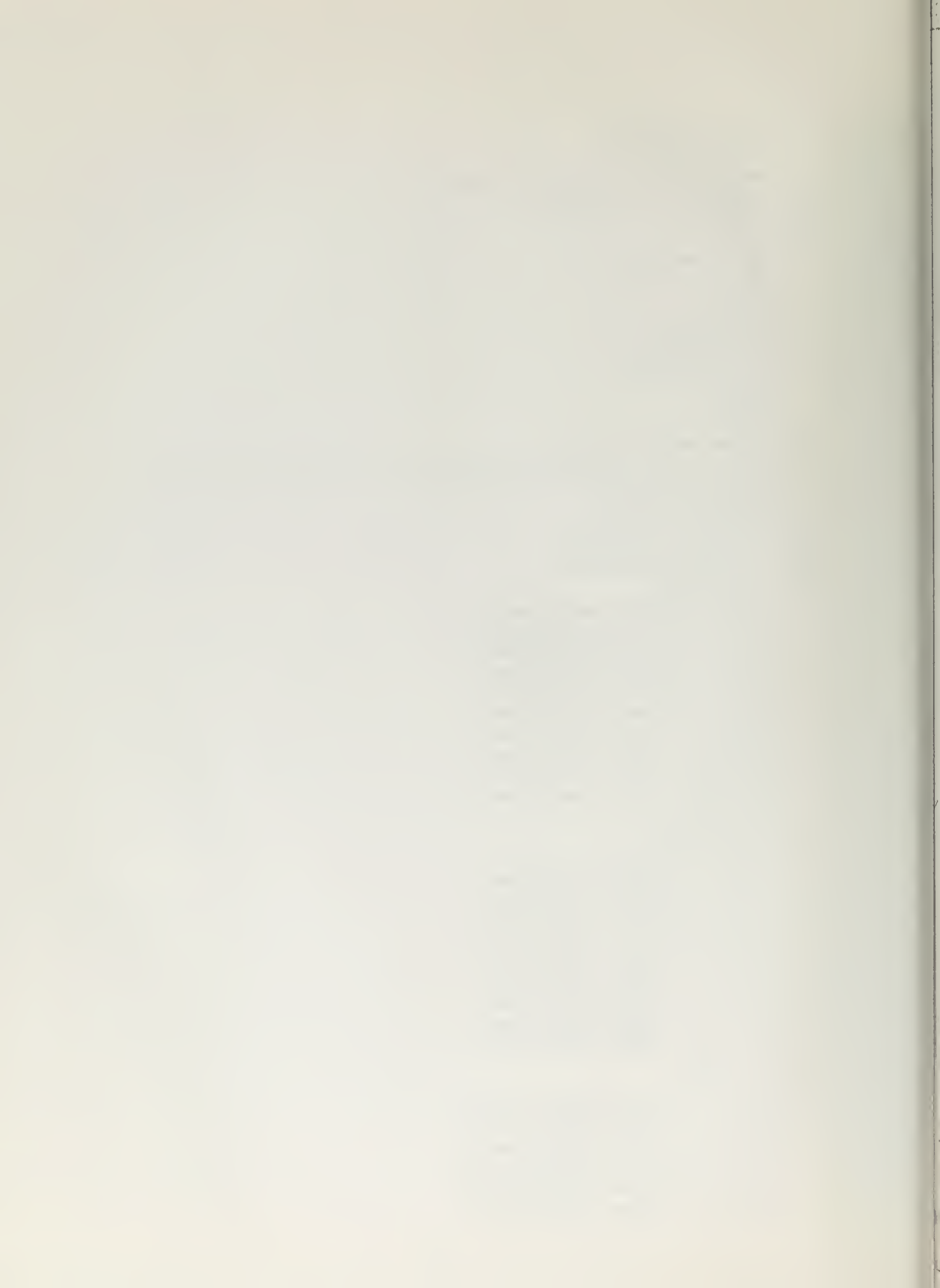
```



```

      U(1,N)=U(11,M)
      U(2,N)=U(21,M)
      T(2,N)=T(21,M)
      PRINT 771, M, U(2,N), T(2,N)
771  FORMAT (I10,2F10.5)
720  CONTINUE
85   L=2
      DO 10 M=2,250
70   XM=M
      Y=0.0
      DELY=0.1
      DELX=0.001
      X(M)=DELY*XM
      I=L+1
      N3N=M
      N=L+1
82   IF(M- 99) 72,73,73
72   READ 1 (U(I,11),U(I,10),U(I,9),U(I,8),U(I,7),U(I,6)
      1,U(I,5),U(I,4),U(I,3),U(I,2) )
      1  FORMAT (10F6.4)
73   IF(M-99 ) 50,51,52
52   IF(M-149) 51,53,54
54   IF(M-199) 53,55,55
51   UU=XM- 99.
      U(N,2) =1.4388+.000740*UU
      U(N,3) =1.4292+.000674*UU
      U(N,4) =1.3993+.000492*UU
      U(N,5) =1.3454+.000238*UU
      U(N,6) =1.2628-.000034*UU
      U(N,7) =1.1467-.000262*UU
      U(N,8) = .9938-.000410*UU
      U(N,9) = .8022-.000452*UU
      U(N,10)= .5720-.000388*UU
      U(N,11)= .3042-.000230*UU
      GO TO 50
53   UU=XM-149.
      U(N,2) =1.4758+.000290*UU
      U(N,3) =1.4629+.000266*UU
      U(N,4) =1.4239+.000194*UU
      U(N,5) =1.3573+.000092*UU
      U(N,6) =1.2611-.000014*UU
      U(N,7) =1.1336-.000104*UU
      U(N,8) = .9733-.000160*UU
      U(N,9) = .7796-.000176*UU
      U(N,10)= .5526-.000150*UU
      U(N,11)= .2926-.000092*UU
      GO TO 50
55   UU=XM-199.
      U(N,2) =1.4903+.0000121*UU
      U(N,3) =1.4762+.0000110*UU
      U(N,4) =1.4336+.0000080*UU
      U(N,5) =1.3619+.0000026*UU
      U(N,6) =1.2604-.0000005*UU
      U(N,7) =1.1284-.0000042*UU

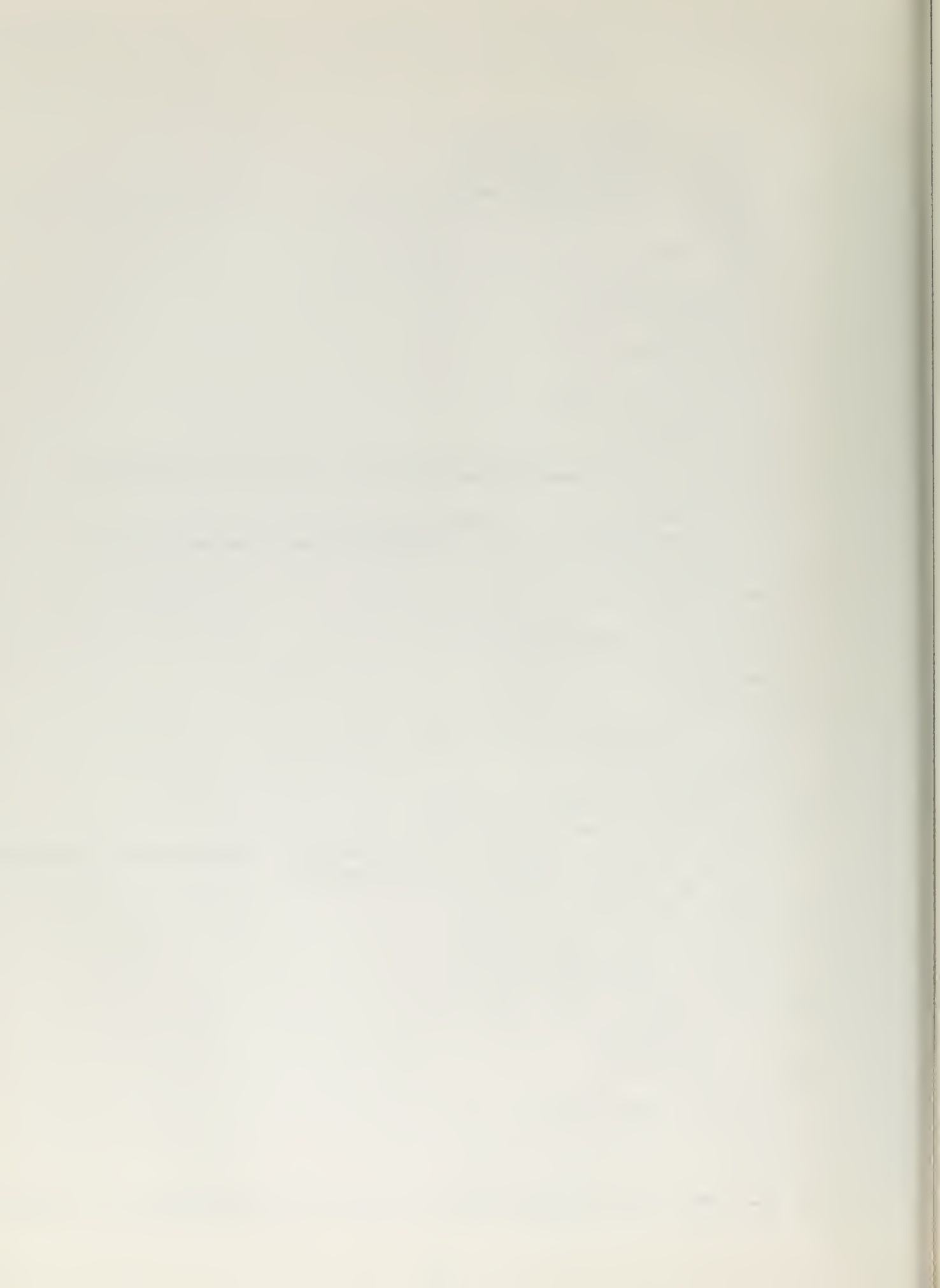
```



```

U(N,8) = .9653-.0000066*UU
U(N,9) = .7708-.0000072*UU
U(N,10) = .5451-.0000063*UU
U(N,11) = .2880-.0000475*UU
50 UTM=0.0
   UM=0.0
   DO 11 J=2,11
     XJ=J
     Y(J)=DELY*XJ-.2
     U(L,12)=0.0
     V(L,2)=0.0
     V(L,1)=V(I,3)
     Y(12)=1.0
     T(L,12)=T(L,11)+0.1
     T(L,1)=T(L,3)
     U(L,1)=U(L,3)
12  V(L,J)= V(L,J-1) + DELY/(2.*DELX*R) * (U(L-1,J)-U(L+1,J))
   A= DELX/ (U(L,J)*PR*DELY**2)
   V(L,12)=0.0
   B= (DELX*R*V(L,J)) / (2.*DELY*U(L,J))
   T(L+1,J)=(A-B)*T(L,J+1)+(1.-2.*A)*T(L,J)+(A+B)*T(L,J-1)
   IF(J-2) 13,13,12
13  UTM=UTM+.5*U(L,J)*T(L,J)
   UM=UM+.5*U(L,J)
   GO TO 14
12  UTM=UTM+U(L,J)*T(L,J)
   UM=UM+U(L,J)
14  TM=UTM/UM
11  CONTINUE
   T1= ( T(L,12)-T(L,11)) / DELY
   T2= T(L,12)-TM
   XN(M)=4.0*T1/T2
   IF(N3N-50 ) 31,31,30
30  IF((N3N/25)*25-N3N) 15,31,31
31  PRINT 18,X(M),XN(M),T2,T1,TM
18  FORMAT (/ 4H X =,F6.3,5X,14H NUSSELT NO. =,F8.5,5X,8H TW-TM =,
19  F8.5,5X,8H SLOPE =,F8.5,5X,5H TM =,F8.5)
   PRINT 19, (T(L,J), J=1,12)
19  FORMAT (/9H TEMP. =,12F7.4)
   PRINT 20, (U(L,J), J=1,12)
20  FORMAT (/ 9H X VEL. =,12F7.4)
   PRINT 21, (V(L,J), J=1,12)
21  FORMAT (/ 9H Y VEL. =,12F7.4)
15  DO 60 J=2,11
     T(L,J)=T(L+1,J)
     U(L-1,J)=U(L,J)
60  U(L,J)=U(L+1,J)
10  CONTINUE
   IF(KK-6) 760,761,761
761 MOD=3
C   GO TO 762
760 MOD=2
C 762 CALL DRAW(497,X,XN,MOD,0,LAB,ITITLE,.04,10.,0,0,0,0,6, 6,1,LAST).
750 CONTINUE

```







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